

Circumference

The circumference of a circle is the distance around the circle. If a circle is compared to a many-sided polygon, the circumference of a circle can be likened to the perimeter of a polygon.

Let us first begin with an exploration regarding circumference.

Consider the table below, which contains a listing of various circular objects (lid, soda can, drinking glass, jar, etc.) In each of these three-dimensional objects, a plane containing a circle has been isolated, and the diameter and circumference of that circle has been listed. Measurements have been rounded to the nearest tenth of a centimeter.

OBJECT	DIAMETER	CIRCUMFERENCE	$\frac{\text{CIRCUMFERENCE}}{\text{DIAMETER}}$
Lid (outer rim)	6.1 cm	19.3 cm	
Soda Can (top rim)	5.5 cm	17.2 cm	
Drinking Glass (top rim)	8.2 cm	25.7 cm	
Jar (top rim)	4.7 cm	14.9 cm	
Mixing Bowl (top rim)	21.6 cm	67.8 cm	
Bucket (top rim)	31.0 cm	97.5 cm	

Exercises

1. Use a calculator to compute the ratio $\frac{\text{Circumference}}{\text{Diameter}}$ for each object in the last column of the table above. Round this ratio to the nearest thousandth.
2. Find three other circular objects. Add them to the table above, measure the circumference and diameter for each, and then compute the ratio $\frac{\text{Circumference}}{\text{Diameter}}$ for each object. (Note: To measure the circumference of an object, you may want to wrap a piece of string around the object, and then measure the string.)
3. Analyze the last column of numbers in the table. What do you notice?

A completed chart can be found below.

OBJECT	DIAMETER	CIRCUMFERENCE	$\frac{\text{CIRCUMFERENCE}}{\text{DIAMETER}}$
Lid (outer rim)	6.1 cm	19.3 cm	3.164
Soda Can (top rim)	5.5 cm	17.2 cm	3.127
Drinking Glass (top rim)	8.2 cm	25.7 cm	3.134
Jar (top rim)	4.7 cm	14.9 cm	3.170
Mixing Bowl (top rim)	21.6 cm	67.8 cm	3.139
Bucket (top rim)	31.0 cm	97.5 cm	3.145

Notice that the numbers in the last column are very close to each other in their numerical value. Ancient mathematicians noticed that the ratio $\frac{\text{Circumference}}{\text{Diameter}}$ always seemed to give the same number, and it became a challenge to try to determine the value of this ratio. Since we rounded to the nearest tenth (and measurement is not completely accurate due to human error and/or measuring devices), the values in our table are not exactly the same. If we were to be able to measure perfectly, we would find that the ratio $\frac{\text{Circumference}}{\text{Diameter}}$ is always equal to the number π (pronounced öpiö).

π is an irrational number, but its value is shown below to 50 decimal places:

$$\pi \approx 3.14159265358979323846264338327950288419716939937510\dots$$

Note: For more information on irrational numbers, see the “Irrational Numbers” tutorial in the Additional Materials section.

Since the ratio $\frac{\text{Circumference}}{\text{Diameter}}$ is equal to π , we can write the following equation:

$$\frac{C}{d} = \pi \quad (\text{Where } C \text{ represents circumference and } d \text{ represents diameter.})$$

Multiplying both sides of the equation by d , we obtain the following equation:

$$C = \pi d$$

Since the diameter d of a circle is twice the radius r , i.e. $d = 2r$, we can also write:

$$C = \pi d = \pi(2r) = 2\pi r$$

Circumference

The circumference of a circle, denoted by C , is the distance around the circle. The circumference is given by either of the following formulas:

$$C = \pi d, \text{ or equivalently, } C = 2\pi r,$$

where d and r represent the diameter and radius of the circle, respectively.