Conditional Probability and Cards

- A standard deck of cards has:
  - 52 Cards in 13 values and 4 suits
  - Suits are **Spades, Clubs**, **Diamonds** and **Hearts**
  - Each suit has 13 card values:
    2-10, 3 “face cards” Jack, Queen, King (J, Q, K) and and Ace (A)

Basic Card Probabilities

- If you draw a card at random, what is the probability you get:
  - A Spade? P(Spade)=13/52
  - A Face card? P(Face Card)=12/52 (or simply 3/13)
  - A Red Ace? P(Red Ace) = 2/52

Multiple Draws without Replacement

- If you draw 3 cards from a deck one at a time what is the probability:
  - All 3 cards are Red?
    - $P(1^{st} \text{ is red} \cap 2^{nd} \text{ is red} \cap 3^{rd} \text{ is red})$
      $= P(1^{st} \text{ is red}) * P(2^{nd} \text{ is red}) * P(3^{rd} \text{ is red})$ by independence
      $= (26/52) * (25/51) * (24/50) = .1176$
  - You don't draw any Spades?
    - $P(1^{st} \text{ isn't Spade} \cap 2^{nd} \text{ isn't Spade} \cap 3^{rd} \text{ isn't Spade})$
      $=P(1^{st} \text{ isn't Spade}) * P(2^{nd} \text{ isn't Spade}) * P(3^{rd} \text{ isn't Spade})$
      $=(39/52) * (38/51) * (37/50) = .4135$

Multiple Draws without Replacement

- If you draw 3 cards from a deck one at a time what is the probability:
  - You draw a Club, a Heart and a Diamond (in that order)
    - P(1st is Club $\cap$ 2nd is Heart $\cap$ 3rd is Diamond)
      = P(1st is Club)*P(2nd is Heart)*P(3rd is Diamond)
      = (13/52) * (13/51) * (13/50) = .0166
  - In any order?
    - There are 6 possible orders (CHD, CDH, DCH, DHC, HCD, HDC) and each is equally likely, so we can multiply .0166 by 6 to get .0996

Independence and Cards

- Are the events “Drawing an Ace” and “Drawing a Red Card” independent?
  - If P(Red Ace)=P(Red)*P(Ace) then yes. Check:
    - P(Red Ace) = 2/52 = 1/26
    - P(Red)*P(Ace)=(1/2) * (1/13) = 1/26
    - Yes, they are independent!