Focusing on Mathematical Reasoning: Transitioning to the 2014 GED® Test

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Activities

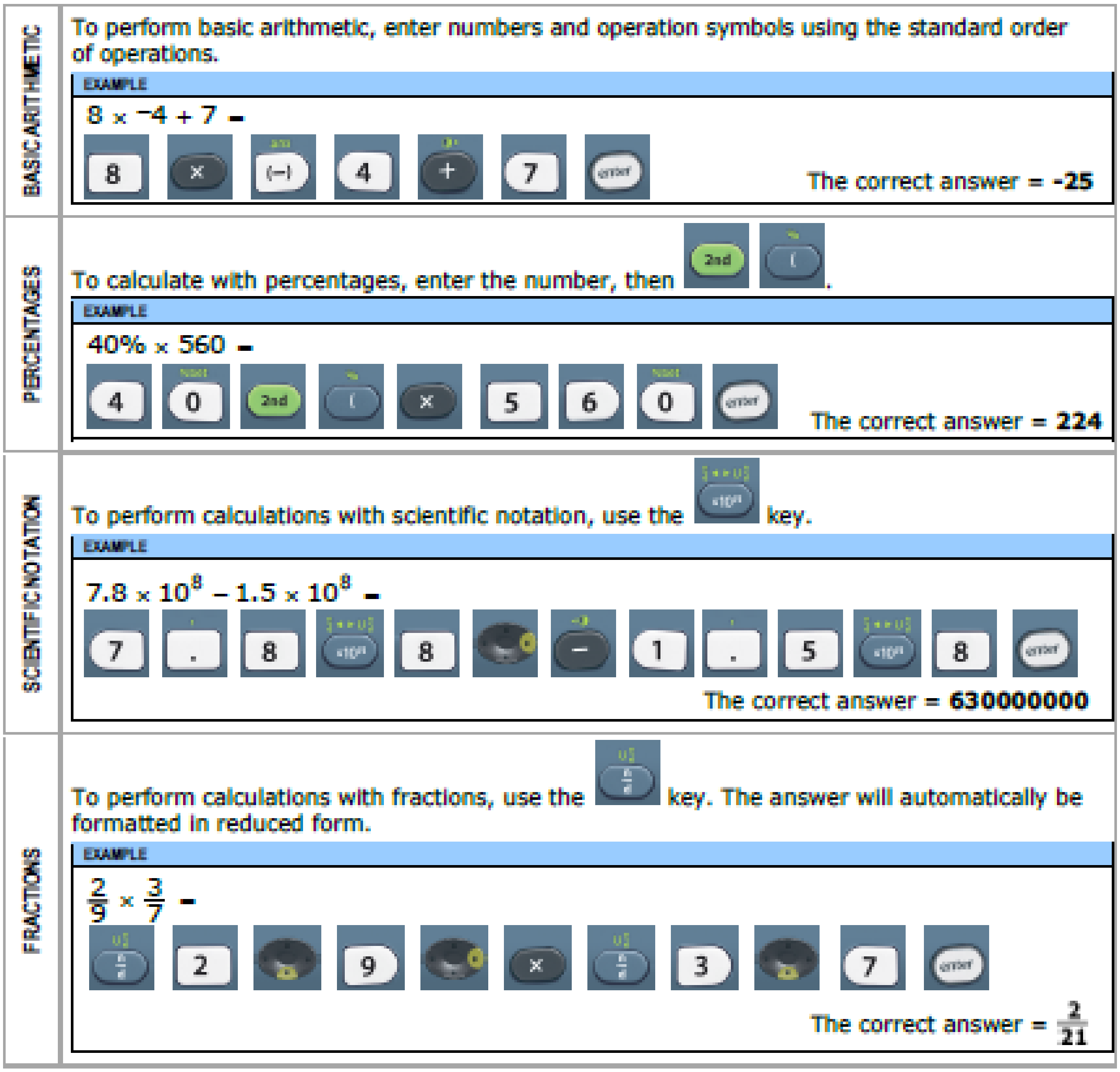
Activity 1 – Can you solve this problem?

10 factories produce sugar cane. The second produced twice as much as the first. The third and fourth each produced 80 more than the first. The fifth produced twice as much as the second. The sixth produced 40 more than the fifth. The seventh and eighth each produced 40 less than the fifth. The ninth produced 80 more than the second. The tenth produced nothing due to drought in Australia. If the sum of the production equaled 11,700, how much did the first factory produce?

Activity 2 – Check Your Skills

TI-30XS MultiView™ Calculator Reference Sheet

The calculator reference sheet is provided on most items on the 2014 GED® Mathematical Reasoning Mathematical Reasoning test, as well as certain items on the Scientific Reasoning and Social Studies tests. The calculator reference sheet is provided to test-takers in order to demonstrate the functionality of the onscreen calculator, specifically in terms of what order to click the buttons in complex problems, such as order of operations or calculating with fractions.



BASIC ARITHMETIC

To perform basic arithmetic, enter numbers and operation symbols using the standard order of operations.

EXAMPLE

$8 \times {}^{-}4 + 7 =$

The correct answer = **-25**

PERCENTAGES

To calculate with percentages, enter the number, then 2nd (.

EXAMPLE

$40\% \times 560 =$

The correct answer = **224**

SCIENTIFIC NOTATION

To perform calculations with scientific notation, use the ×10ⁿ key.

EXAMPLE

$7.8 \times 10^8 - 1.5 \times 10^8 =$

The correct answer = **630000000**

FRACTIONS

To perform calculations with fractions, use the fraction key. The answer will automatically be formatted in reduced form.

EXAMPLE

$\frac{2}{9} \times \frac{3}{7} =$

The correct answer = $\frac{2}{21}$

MIXED NUMBERS

To perform calculations with mixed numbers, use .

As with fractions, the answer will automatically be formatted in reduced form.

EXAMPLE

$12\frac{5}{6} - 1\frac{1}{2} =$

The correct answer = $\mathbf{\frac{34}{3}}$

POWERS AND ROOTS

To perform calculations with powers and roots, you will use the following keys:

EXAMPLE

$1.2^2 =$

The correct answer = **1.44**

EXAMPLE

$7^4 =$

The correct answer = **2401**

EXAMPLE

$\sqrt{529} =$

The correct answer = **23**

EXAMPLE

$\sqrt[3]{1728} =$

The correct answer = **12**

TOGGLE KEY

The answer toggle key  can be used to toggle the display result between fraction and decimal answers, exact square root and decimal, and exact pi and decimal.

EXAMPLE

$\frac{9}{10} =$

The correct answer = **0.9**

Activity 3 – Problem-Solving Process (adapted from Polya)

Step 1: Understanding the Problem			
1. Read/Reread (for understanding)	2. Paraphrase (your own words)	3. Visualize (mentally or drawing	4. Work in pairs or small groups
5. Identify goal or unknown	6. Identify required information	7. Identify extraneous information	8. Detect missing information
9. Define/Translate Use a dictionary	10. Check conditions and/or assumptions	11. Share point of view with others	12. Others as needed
Step 2: Devising a Plan to Solve the Problem			
1. Estimate (quantity, measure, or magnitude)	2. Revise 1st estimate, 2nd estimate, etc.	3. Share/discuss strategies	4. Work in pairs or small groups
5. Explain why the plan might work	6. Each try a common strategy or a different one	7. Reflect on Possible Solution Processes	8. Others as needed
Step 3: Implementing a Solution Plan			
1. Experiment with different solution plans	2. Allow for mistakes/errors	3. Show all work, including partial solutions	4. Work in pairs or small groups
5. Discuss with others different solution plans	6. Keep track and save all results/data	7. Compare attempts to solve similar problems	8. Find solution – do not give up
9. Implement solution plan	10. Realize attempts can be as important as the solution	11. Check the answer(s) and solution(s)	12. Others as needed
Step 4: Reflection on the Problem: Looking Back			
1. Reflect on plan after answer is obtained	2. Reflect on plan while finding the answer	3. Check if all problem conditions were made	4. Make sure the answer can be justified/ explained
5. Check if correct assumptions were made	6. Check that the solution answers the problem question	7. Check if answer is unique or if there are others	8. Reflect for possible alternative strategies
9. Reflect about possible, more efficient process	10. Look for ways to extend the problem	11. Reflect on similarity and/or difference to other problems	12. Others as needed

Activity 4 – It's Your Turn to SOLVE

Mathematical Reasoning - Candidate Name

Question 7 of 16

Answer Explanation Calculator Flag for Review

Formula Sheet Calculator Reference

Type your answer in the box. You may use numbers, a decimal point (.), and/or a negative sign (-) in your answer.

Hartley opened a food truck business to sell food on the street. On day 2, the business earned $112. On day 5, the business earned $367. Hartley assumes that the earnings will continue to increase at the same rate. How much will the business earn on day 10?

$ [ ]

S

O

L

V

E

Activity 5 – It's Your Turn to SOLVE

Mathematical Reasoning - Candidate Name

Question 16 of 16

Answer Explanation | Calculator | Flag for Review

Formula Sheet | Calculator Reference

Type your answer in the box. You may use numbers, a decimal point (.), and/or a negative sign (-) in your answer.

The probability of rain on each of the next three days is given in the table.

Probability of Rain on Each Day

Day	Tuesday	Wednesday	Thursday
Probability of Rain	30%	45%	50%

Based on the table, what is the percent probability that it will rain all three days?

[ ] %

S

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L

V

E

Activity 6 – Can You See It?

Visualization 1

Visualization 2

Visualization 3

Visualization 4

Visualization 5

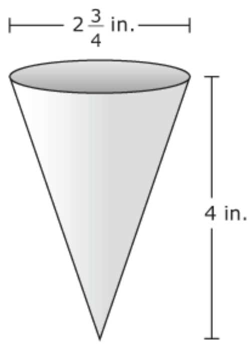
Activity 7 – A Return to SOLVE

Mathematical Reasoning - Candidate Name   Question 12 of 16

Answer Explanation   Calculator   Flag for Review

Formula Sheet   Calculator Reference

An office uses paper drinking cups in the shape of a cone, with dimensions as shown.



To the nearest tenth of a cubic inch, what is the volume of each drinking cup?

- A. 2.5
- B. 7.9
- C. 23.7
- D. 31.7

S

O

L

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E

Activity 8 – Time Out to Determine the Digit!

In this number puzzle, each letter (q - z) represents a different digit from 0-9. Find the correspondence between the letters and the digits. Be prepared to explain where you started, and the order in which you solved the puzzle.

1. $u \bullet r = z$
2. $t + w = t$
3. $r + r + r + r = z$
4. $x + y = q$
5. $s \bullet v = s$
6. $x^2 = q$
7. $r + r = u$
8. $x + u = s$
9. $\frac{y}{z} = \frac{x}{u}$

Activity 9 – Algebra Tile Mat

x

Activity 10 – Best Practices Review

Instructional Element	Recommended Practices
Curriculum Design	• Ensure mathematics curriculum is based on challenging content • Ensure curriculum is standards based • Clearly identify skills, concepts and knowledge to be mastered • Ensure that the mathematics curriculum is vertically and horizontally articulated

Professional Development for Teachers	• Provide professional development which focuses on:
	  ○ Knowing/understanding standards
	  ○ Using standards as a basis for instructional planning
	  ○ Teaching using best practices
	  ○ Multiple approaches to assessment
	• Develop/provide instructional support materials such as curriculum maps and pacing guides and provide math coaches

Technology	• Provide professional development on the use of instructional technology tools • Provide student access to a variety of technology tools • Integrate the use of technology across all mathematics curricula
Manipulatives	• Use manipulatives to develop understanding of mathematical concepts • Use manipulatives to demonstrate word problems • Ensure use of manipulatives is aligned with underlying math concepts
Instructional Strategies	• Focus lessons on specific concept/skills that are standards based • Differentiate instruction through flexible grouping, individualizing lessons, compacting, using tiered assignments, and varying question levels • Ensure that instructional activities are learner-centered and emphasize inquiry/problem-solving • Use experience and prior knowledge as a basis for building new knowledge • Use cooperative learning strategies and make real life connections • Use scaffolding to make connections to concepts, procedures and understanding • Ask probing questions which require students to justify their responses • Emphasize the development of basic computational skills
Assessment	• Ensure assessment strategies are aligned with standards/concepts • Evaluate both student progress/performance and teacher effectiveness • Utilize student self-monitoring techniques • Provide guided practice with feedback • Conduct error analyses of student work • Utilize both traditional and alternative assessment strategies • Ensure the inclusion of diagnostic, formative and summative strategies • Increase use of open-ended assessment techniques

Research and Articles

Teaching Mathematics: Connect It to the Real World!

Students are often fearful regarding mathematics. Address and evaluate attitudes and beliefs regarding both learning math and using math. Prior to any true learning taking place, the instructor must discuss with students how traditional methods of teaching mathematics may have caused them to develop a negative attitude.

1. Determine what students already know about a topic before instruction. Use an informal discussion of what students already know about a topic prior to teaching. Formal assessment instruments do not always provide an accurate picture of a student's real life knowledge or thinking processes. For example, if discussing positive and negative integers, discuss a bank account and the concept of being "overdrawn" or in the negative category.

2. Develop understanding by providing opportunities to explore mathematical ideas with concrete or visual representations and hands-on activities. Students will learn more effectively if they can visualize concretely an abstract concept (If you can “see” it you can solve it).

3. Use manipulatives such as Cuisenaire rods, fraction circles, geoboards, algebra tiles, or everyday objects such as coins, toothpicks, etc. to help students explain how mathematical rules and concepts work. Start with concrete objects to move to abstract ideas.

4. Encourage the development and practice of estimation skills. During everyday life, one does not always use "exact" math. Teach students how to estimate. Strategies to use can include rounding to whole numbers, multiplying by 10 rather than 9, or dividing by whole numbers rather than multiplying by fractions. Use test examples to show students that good estimation can result in correct answers. Have the students work out the problem using computation skills to support their estimations.

5. Emphasize the use of "mental math" as a legitimate alternative computational strategy. Encourage the development of mental math skills by making connections between different mathematical procedures and concepts. GED students often have difficulty with multi-step problems. Teach them mental math strategies that "make sense to them." An example would be to multiply $4.00 by 4 and then subtract 4 from the answer to solve the equation $3.99 x 4. Utilizing division rather than percentages is another example of mental math, i.e. dividing a number by 4 rather than multiplying it by the equivalent percentage 25%. Always discuss the "why" of such skills and whether paper/pencil calculation or mental math is the best method to use for the problem.

6. View computation as a tool for problem solving, not an end in itself. GED students must learn more than mere calculations. The GED Tests focus on *why* and *when* concepts. Integrate problem-solving abilities while teaching computation skills. Story problems or real-life problems must be a significant part of instructional time. Have students write

their own story problems to reinforce the connection between computation and real-life skills. Share the problems with the class, providing a mixture of computational procedures rather than supporting just one concept such as subtracting fractions.

7. Encourage use of multiple solution strategies. Teach your students how to solve problems in different ways. In many mathematical problems, there is more than one way to find the solution. There are eight categories of problem solving strategies. Help students learn how to use as many of these as possible, so they have a full repertoire of strategies to draw from when they encounter different problems. These categories of problem-solving strategies include:
   - Drawing a picture or diagram
   - Marking a chart or graph
   - Dividing a problem into smaller parts
   - Looking for patterns
   - Using a formula or written equation
   - Computing or simplifying
   - Using the process of elimination
   - Working backwards

8. Help students understand the process required in problem solving.
   - Read the problem carefully. Reread if necessary.
   - Determine the meaning of key words or special terms.
   - State the goal in your own words.
   - List the important information.
   - Recall similar problems and recall how they were solved.
   - Try different approaches.
   - Match your solution with the original goal. Does it make sense? Is it accurate?

9. Develop students' calculator skills. Calculators are allowed on Part I of the GED Mathematics Test. Since calculators are allowed, set aside time to teach students how to correctly use the calculators to perform single and multi-tasked problems. Calculators can be used by students to check their work, to solve tedious computations, and as a problem-solving tool.

10. Provide opportunities for group work. Develop a project where a group effort is appropriate. An example would be to organize an activity where the development of a plan, schedule, budget, needed business materials, and a report would be required. As with all group activities, clear goals and rules must initiate the project. A rubric would be helpful in providing students with the structure to assess their own progress as a group.

11. Link numeracy and literacy instruction by providing opportunities for students to communicate about mathematical issues. Students need to be able to communicate about math. Teachers should provide real life activities in which math is used. Examples

would include teaching math concepts to others, letters of complaint to companies clearly detailing a billing problem, a detailed explanation of why a bank statement was incorrect, or a formal discussion of why a method was selected to solve a particular real life problem.

12. Provide problem-solving tasks within a meaningful, realistic context in order to facilitate transfer of learning.

13. Students need to view math as a necessary skill in their lives. Students can assist transference of mathematical skills to real-life experiences through the sharing of experiences. These experiences can be used as problem-solving projects for the class. Projects can be as simple as comparing the price of cereals to as complex as finding the best mortgage deal. Discuss how students use math in their daily lives and set up problems based upon these scenarios.

14. Develop students' skills in interpreting numerical or graphical information appearing within documents and text. Math does not always take the form of computation. Graphs, tables, text, payment schedules, and contracts are just a few of the ways in which text is filled with mathematical concepts. Strategies to use in teaching students how to accurately interpret such documents can include having students graph information from their lives for the last 24 hours. Pictorial, circle, line, or any type of graph can be used to visually document numerical information. Another activity would be to have students critique and discuss an article filled with numerical information such as an employee benefit statement.

15. Assess a broad range of skills, reasoning processes, and dispositions using a range of methods. The GED® Mathematics module assesses skills in a variety of methods. Use various assessment strategies when teaching mathematics. Include not only multiple-choice questions, but also alternate format questions that require the use of standard and coordinate plane grids. Provide open-ended questions, graphs, tables, text, and short answer questions requiring verbal documentation of how a solution was obtained. Remember that assessments do not always have to be formally graded. Having students write down the steps they use to solve a problem and why they chose those particular steps can help students become more confident in their problem solving ability. Written explanations can also offer you insight into areas that are causing problems and where students may need additional instruction.

Why Study Geometry?

Improving students' geometric thinking levels is one of the major aims of mathematics education. Geometric thinking is very important in scientific, technical, and occupational areas, as well as in mathematics. So what is geometry? Geometry is the attempt to understand space, shape, and dimension. It's about the properties of objects (their angles and surfaces, for instance) and the consequences of how these objects are positioned (where their shadows fall, how people must move around them).

When adults think about geometry, they usually remember a course in the second year of high school—one loaded with proofs about isosceles triangles and vertical angles. For many of us, the geometry course sounded the death knell for our progress—and interest—in mathematics. Geometry, however, is not abstract. Geometry is naturally concrete. It's fun and colorful, instructive and practical. Geometry is about real things: how big they are, whether they fit, how to find them, what they look like in a mirror.

Spatial understandings are necessary for interpreting, understanding, and appreciating our inherently geometric world. Insights and intuitions about two- and three-dimensional shapes and their characteristics, the interrelationships of shapes, and the effects of changes to shapes are important aspects of spatial sense. Students who develop a strong sense of spatial relationships and who master the concepts and language of geometry are better prepared to learn number and measurement ideas, as well as other advanced mathematical topics.

Spatial visualization has everyday application. It's not just preparation for the abstract geometry class, but rather geometry for everyday life. If students state they are not very good at this kind of thing, be assured, it can be learned. Students do improve with practice.

The Development of Geometric Thinking

Geometry curriculum is often presented through the memorization and application of formulas, axioms, theorems, and proofs. This type of instruction requires that students function at a formal deductive level. However, many students lack the prerequisite skills and understanding of geometry in order to operate at this level.

The work of two Dutch educators, Pierre van Hiele and Dina van Hiele-Geldof, are having a major impact on the design of geometry instruction and curriculum. The van Hiele's work began in 1959 and has since become the most influential factor in the American geometry curriculum. The van Hiele model is a five-level hierarchy of ways of understanding spatial ideas. Each of the five levels describes the thinking processes used in geometric contexts. The levels describe how one thinks and what types of geometric ideas one thinks about, rather than how much knowledge one has.

The van Hiele levels are not age dependent. A well-crafted geometry lesson should be accessible to all students and should allow students to work at their own level of development. The levels are sequential in nature so that as students progress from one level to another, their geometric thinking changes.

Level 1: Visualization

The objects of thought at level 1 are shapes and what they "look like."

Students recognize and name figures based on the global, visual characteristics of the figure—a gestalt-like approach to shape. Students operating at this level are able to make measurements and even talk about properties of shapes, but these properties are not thought about explicitly. It is the appearance of the shape that defines it for the student. A square is a square "because it looks like a square." Because appearance is dominant at this level, appearances can overpower properties of a shape. For example, a square that has been rotated so that all sides are at a 45° angle to the vertical may not appear to be a square for a level 1 thinker. Students at this level will sort and classify shapes based on their appearances—"I put these together because they all look sort of alike."

The products of thought at level 1 are classes or groupings of shapes that seem to be "alike."

Level 2: Analysis

The objects of thought at level 2 are classes of shapes rather than individual shapes.
Students at the analysis level are able to consider all shapes within a class rather than a single shape. Instead of talking about *this* rectangle, it is possible to talk about *all* rectangles. By focusing on a class of shapes, students are able to think about what makes a rectangle a rectangle (four sides, opposite sides parallel, opposite sides same length, four right angles, congruent diagonals, etc.). The irrelevant features (e.g., size or orientation) fade into the background. At this level, students begin to appreciate that a collection of shapes goes together because of properties. Ideas about an individual shape can now be generalized to all shapes that fit that class. If a shape belongs to a particular class such as cubes, it has the corresponding properties of that class. "All cubes have six congruent faces, and each of those faces is a square." These properties were only implicit at level 0. Students operating at level 2 may be able to list all the properties of squares, rectangles, and parallelograms but not see that these are subclasses of one another that all squares are rectangles and all rectangles are parallelograms. In defining a shape, level 2 thinkers are likely to list as many properties of a shape as they know.

The products of thought at level 2 are the properties of shapes.

Level 3: Informal Deduction

The objects of thought at level 3 are the properties of shapes.
As students begin to be able to think about properties of geometric objects without the constraints of a particular object, they are able to develop relationships between and among these properties. "If all four angles are right angles, the shape must be a rectangle. If it is a square, all angles are right angles. If it is a square, it must be a rectangle." With greater ability to engage in "if-then" reasoning, shapes can be classified using only minimum characteristics. For example, four congruent sides and at least one right angle can be sufficient to define a square. Rectangles are parallelograms with a right angle. Observations go beyond properties themselves and begin to focus on logical arguments *about* the properties. Students at level 3 will be able to follow and appreciate an informal deductive argument about shapes and their properties. "Proofs" may be more intuitive than rigorously deductive. However, there is an appreciation that a logical argument is compelling. An appreciation of the axiomatic structure of a formal deductive system, however, remains under the surface.

The products of thought at level 3 are relationships among properties of geometric objects.

Level 4: Deduction

The objects of thought at level 4 are relationships among properties of geometric objects.

At level 4, students are able to examine more than just the properties of shapes. Their earlier thinking has produced conjectures concerning relationships among properties. Are these conjectures correct? Are they “true”? As this analysis of the informal arguments takes place, the structure of a system complete with axioms, definitions, theorems, corollaries, and postulates begins to develop and can be appreciated as the necessary means of establishing geometric truth. At this level, students begin to appreciate the need for a system of logic that rests on a minimum set of assumptions and from which other truths can be derived. The student at this level is able to work with abstract statements about geometric properties and make conclusions based more on logic than intuition. This is the level of the traditional high school geometry course. A student operating at level 4 can clearly observe that the diagonals of a rectangle bisect each other, just as a student at a lower level of thought can. However, at level 4, there is an appreciation of the need to prove this from a series of deductive arguments. The level 3 thinker, by contrast, follows the argument but fails to appreciate the need.

The products of thought at level 4 are deductive axiomatic systems for geometry.

Level 5: Rigor

The objects of thought at level 5 are deductive axiomatic systems for geometry.
At the highest level of the van Hiele hierarchy, the object of attention is axiomatic systems themselves, not just the deductions within a system. There is an appreciation of the distinctions and relationships between different axiomatic systems. This is generally the level of a college mathematics major who is studying geometry as a branch of mathematical science.

The products of thought at level 5 are comparisons and contrasts among different axiomatic systems of geometry.

Teaching Geometric Thinking: A Few Strategies

Research has shown that the use of the following strategies is effective in assisting students to learn concepts, discover efficient procedures, reason mathematically, and become better problem solvers in the areas of geometry and measurement.

- Have high expectations for all students
- Base practice on educational research
- Integrate content areas
- Encourage cooperative learning and collaboration with others
- Use technology as a tool
- Use inquiry-based learning
- Promote mathematical reasoning and problem-solving skills
- Use hand-on activities to model topics
- Cluster concepts, rather than teaching concepts in isolation
- Reflect and communicate with others on what is being learned
- Integrate assessment and instruction

Algebraic Thinking

What is algebraic thinking? When do you think students first begin to algebraically think? Algebraic thinking is very simply the ideas of algebra and the skill of being able to logically think. Algebraic ideas include patterns, variables, expressions, equations, and functions. These are the building blocks of algebraic thinking. Translating words into symbols is similar to modeling a situation using an equation and variables. Students need to know that it is through algebraic equations and inequalities that they can represent a quantitative relationship between two or more objects.

Teaching algebra in today's classroom is not as much about manipulating letters and numbers in equations that don't make sense, but rather understanding operations and processes. Before beginning the process of teaching algebra, be sure that students understand the basics. The key prerequisites for students to be successful in the study of algebra are to first understand the:

- concept of variables; and
- concept of relations and functions.

When teaching algebra, it's important to use practical experiences that go beyond the mere computation required by equations. When developing practice activities in the algebra classroom, be sure that you:

- Develop processes/procedures for students to use when approaching algebraic tasks
- Create exercises that highlight the critical attributes related to the skill or concept being taught
- Provide opportunities for students to verbalize about the task and predict what type of answer is expected
- Offer opportunities for students to discuss and write responses to questions dealing with key concepts being learned
- Select exercises that anticipate future skills to be learned
- Design exercises that integrate a number of ideas to reinforce prior learning as well as current, and future concepts

As students learn algebra, they need to develop different procedures to use. Being able to recognize a pattern is an important critical thinking skill in solving certain algebraic problems.

- **Finding** patterns involves looking for regular features of a situation that repeats.
- **Describing** patterns involves communicating the regularity in words or in a mathematically concise way that other people can understand.
- **Explaining** patterns involves thinking about why the pattern continues forever, even if one has not exhaustively looked at each one.
- **Predicting** with patterns involves using your description to predict pieces of the situation that are not given.

Components of Algebraic Thinking	
Mathematical Thinking Tools Problem solving skills • Using problem solving strategies • Exploring multiple approaches/multiple solutions Representation skills • Displaying relationships visually, symbolically, numerically, verbally • Translating among different representations • Interpreting information within representations Reasoning skills • Inductive reasoning • Deductive reasoning	Informal Algebraic Ideas Algebra as abstract arithmetic • Conceptually based computational strategies • Ratio and proportion Algebra as the language of mathematics • Meaning of variables and variable expressions • Meaning of solutions • Understanding and using properties of the number system • Reading, writing, manipulating numbers and symbols using algebraic conventions • Using equivalent symbolic representations to manipulate formulas, expressions, equations, inequalities Algebra as a tool to study functions and mathematical modeling • Seeking, expressing, generalizing patterns and rules in real-world contexts • Representing mathematical ideas using equations, tables, graphs, or words • Working with input/output patterns • Developing coordinate graphing skills

Math Journals in the GED® Classroom

Writing activities can help students better understand the material they are trying to learn and ultimately can shift students from looking at math as a series of formulas that have to be solved or computations that must be completed to recognizing that mathematics is a process. Most GED® students do not recognize that mathematics is a process; rather, they see each problem with a specific answer and no real relationship among the wide range of problems that they encounter in the classroom, on tests, or in the real world.

Math journals can be used for many purposes. The GED® teacher should look at math journals as variables rather than constants, providing opportunities for students to:

- Increase their feelings of confidence in being able to learn and use mathematical concepts and skills to solve a wide range of problems and thus help alleviate math anxiety.
- Be more aware of what they do and do not know.
- Make use of their own prior knowledge when solving new problems.
- Identify their own questions about an area with which they are less familiar.
- Develop their ability to think through a problem and identify possible methods for solving it.
- Collect and organize their thoughts.
- Monitor their own progress as they gain higher-level problem-solving skills and are able to work with more complex problems.
- Make connections between mathematical ideas as they write about various strategies that could be used for problem solving.
- Communicate more precisely how they think.

In *Writing in the Mathematics Curriculum* (Burchfield, Jorgensen, McDowell, and Rahn 1993), the authors identify three possible categories for math journal prompts. These categories include:

- Affective/attitudinal prompts, which focus on how students feel.
- Mathematical content prompts, which focus on what the material is about.
- Process prompts, which require students to explain what they are thinking and doing.
- Using Affective/Attitudinal Prompts in Math Journals

Many adult learners are math phobic or, at least, fearful of trying and failing to solve problems. Their own feelings of inability to learn mathematics get in their way and, in essence, become a self-fulfilling prophecy. The more anxious the learner becomes, the less he/she is able to focus on the math content. Affective/attitudinal math journal prompts enable students to express their feelings, concerns, and fears about mathematics.

The following are a few examples of affective/attitudinal prompts:

- Explain how you feel when you begin a math session.
- One secret I have about math is…
- If I become better at math, I can…
- My best experience with math was when…

- My worst experience with math was when…
- Describe how it feels if you have to show your work on the board…
- One math activity that I really enjoyed was…

Using Mathematical Content Prompts in Math Journals

When working with math content, most adult learners expect merely to perform a series of computations and provide a specific answer. Rarely have they been asked to explain what they did to find an answer. Mathematical content prompts provide learners with an opportunity to explain how they arrived at a specific answer, thus enabling them to begin making connections between what they have done and the math content itself. These types of prompts also enable students to support their point of view or to explain errors they made in their calculations.

Mathematical content prompts can be as simple as students writing definitions in their own terms, such as defining geometric shapes or providing math examples of what variables are and why they are used.

The following are a few examples of mathematical content prompts:

- The difference between … and … is…
- How do you…?
- What patterns did you find in…?
- How do you use … in everyday life?
- Explain in your own words what … means.
- One thing I have to remember with this kind of problem is…
- Why do you have to…?

Using Process Prompts in Math Journals

Process prompts allow learners to explore how they go about solving a problem. It moves them from mere computations to looking at math problem solving as a process that, just as in solving real-life problems, requires a series of steps and questions that must be analyzed and answered.

Process prompts require learners to look more closely at how they think.

The following are examples of process prompts:

- How did you reach the answer for the problem about…?
- What part in solving the problem was the easiest? What was the most difficult? Why?
- The most important part of solving this problem was…
- Provide instructions for a fellow student to use to solve a similar problem.
- What would happen if you missed a step in the problem? Why?
- What decisions did you have to make to solve this type of problem?
- When I see a word problem, the first thing I do is…
- Review what you did today and explain how it is similar to something you already knew.
- Is there a shortcut for finding…? What is it? How does it work?
- Could you find the answer to this problem another way?

- I draw pictures or tables to solve problems because…
- To solve today's math starter, I had to…
- The first answer I found for this problem was not reasonable, so I had to…

Material Adapted From

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Whitin, Phyllis and Whitin, David J. (2000). *Math Is Language Too: Talking and Writing in the Mathematics Classroom.* Urbana, IL: National Council of Teachers of English, and Reston, VA: National Council of Teachers of Mathematics.

Classroom Questioning

Questions are central to the understanding. When asking questions:

- Ask clear specific questions. If students have to guess at what you are asking, they are likely to remain quiet and not engage in the thinking you are expecting.
- Use cueing vocabulary that is familiar to students. By using the vocabulary with which they are familiar, students can better answer the question.
- Ask follow up questions to get at students' real understandings. Asking a second question can reveal the difference between a student's accurate understanding and misconceptions.
- Remember "wait time." Provide at least five seconds of thinking time after a question and after a response. Students need time to think and organize their response. Waiting lets students know that you are serious about wanting an answer to your question.
- Create a climate that supports risk taking. Establish eye contact and withhold judgment. Let students know that there is not a single correct answer for some questions.
- Allow students to ask their own questions. This often will further develop a topic and let students know you are interested in their reasoning.
- Listen to the answer.

Materials for the Classroom

Can You Remember?

This activity tests your memory of familiar geometric shapes. Draw the shapes and then compare your drawing to the actual size of the real items. You should try and draw them as realistically as possible. Use another sheet of paper for your drawings.

1. Draw circles the size of a penny, nickel, dime, and quarter.

2. Draw a circle the size of a CD.

3. Draw a circle the size of the bottom of a soda can.

4. Draw a rectangle the size of a dollar bill.

5. Draw a line that is the length of a computer mouse.

6. Draw a square the size of a key on a computer keyboard.

7. Draw a line that is the length of your foot.

8. Draw a line that is the length of one joint of your index finger.

9. Draw a rectangle the size of a credit card.

10. Draw a rectangle the size of a paperback book.

11. Draw a line the length of your house key.

12. Draw a line the length of a fork.

13. Draw a rectangle the size of a large paper clip.

14. Draw an oval the size of an egg.

15. Draw a rectangle the size of a business card.

Sample Journal Prompts to Get You Started

Affective/Attitudinal Prompts

Explain how you feel when you begin a math session.

One secret I have about math is . . .

If I become better at math, I can . . .

My best experience with math was when . . .

My worst experience with math was when . . .

Describe how it feels if you have to show your work on the board . . .

One math activity that I really enjoyed was . . .

People who are good at math are . . .

My three personal goals for this course are . . .

Content Prompts

The difference between . . . and . . . is . . .

How do you . . . ?

What patterns did you find in . . . ?

How do you use . . . in everyday life?

Explain in your own words what . . . means.

One thing I have to remember with this kind of problem is . . .

Why do you have to . . . ?

Describe practical uses for . . .

Compare and contrast . . .

Process Prompts

How did you reach the answer for the problem about . . . ?

What part in solving the problem was the easiest? What was the most difficult? Why?

The most important part of solving this problem was . . .

Provide instructions for a fellow student to use to solve a similar problem.

What would happen if you missed a step in the problem? Why?

What decisions did you have to make to solve this type of problem?

When I see a word problem, the first thing I do is . . .

Review what you did today and explain how it is similar to something you already knew.

Is there a shortcut for finding . . . ? What is it? How does it work?

Could you find the answer to this problem another way?

I draw pictures or tables to solve problems because . . .

To solve today's math starter, I had to . . .

The first answer I found for this problem was not reasonable, so I had to . . .

Material Adapted From
Math Journals for All Ages. Retrieved from the World Wide Web at: http://math.about.com/aa123001a.htm.

Burchfield, P.C., Jorgensen, P.R., McDowell, K.G., and Rahn, J. (1993). *Writing in the Mathematics Curriculum.* Retrieved from the World Wide Web at: http://www.geocities.com/kaferico/writemat.htm.

Countryman, J. (1992). *Writing to Learn Mathematics.* Portsmouth, NH: Heinemann.

Whitin, Phyllis and Whitin, David J. (2000). *Math Is Language Too: Talking and Writing in the Mathematics Classroom.* Urbana, IL: National Council of Teachers of English, and Reston, VA: National Council of Teachers of Mathematics.

Sample Questions for the Classroom

Questions for a Student Interview: Understanding Mathematical Processes

- What does this mean?
- What are doing here? (indicating something on student work)
- Tell me where you're getting each of your numbers from here.
- Why did you decide to ... ?
- I don't understand. Could you show me an example of what you mean?
- So what are you going to try next?
- What are you thinking about?
- Is there another idea you might try?
- Why did you decide to begin with ... ?
- Do you have any ideas about how you might figure out ... ?
- You just wrote down ____. Tell me how you got that.
- What are you doing there with those numbers?

Phrases to Avoid in a Student Interview

- Let me show you how to do this.
- That's not correct.
- I'm not sure you want to do that. / Why on earth would you do *that*!?

Vocabulary Match!

Coefficient	a constant that is being multiplied by a variable or by another expression	7n in the expression 7(n+42)
Constant	to remain the same	n in the expression 37 + n
Equation	two equal values	36 X 14 = 504
Exponent	a number that tells how many times the base (of a power) is written in the product	The ^{2} in x^2
Expression	a mathematical/algebraic phrase	36 X 14, 2x - y
Inequality:	compares two values that may or may not be equal	36 X 14 > 500
Integers	all positive and negative counting numbers including 0	A number from the set {… -3, -2, -1, 0, 1, 2, 3 …}
Terms	parts of an expression or series separated by + or – signs, or parts of a sequence	$2a^3 – 6$ or $5a^3$, $2xy$, and 3
Solution	replaces the variable to produce a true equation	n + 19 = 21, n = 2
Variable	a letter in place of a number, the value will be different in different equations	2x + 4, 4y-2
Monomial	an expression consisting of a single term	x, 2y
Binomial	an expression consisting of 2 terms connected by a plus or minus sign	2x + 4, 4y-2
Trinomial	an expression consisting of 3 terms	2x + 4y - z
Polynomial	an expression of one or more algebraic terms of which consists of a constant multiplier by one or more variables raised to a power	$3x^2 + 4x + 5$

Discovering Exponents

For each pair of exponents listed below determine which is greater. First guess and then check your answer using the calculator, x^2 or y^x key.

	Guess	Check
1. 2^3 or 3^2		
2. 4^5 or 5^4		
3. 6^2 or 2^6		
4. 8^9 or 9^8		
5. 7^9 or 9^7		
6. 5^8 or 8^5		
7. 3^9 or 9^3		
8. 3^4 or 4^3		
9. 2^5 or 5^2		
10. 1^7 or 7^1		
11. 3^0 or 0^3		
12. 2^{-5} or 5^{-2}		
13. 9^{-3} or 3^{-9}		
14. 0^1 or 1^0		

Write five statements about your findings.

1.

2.

3.

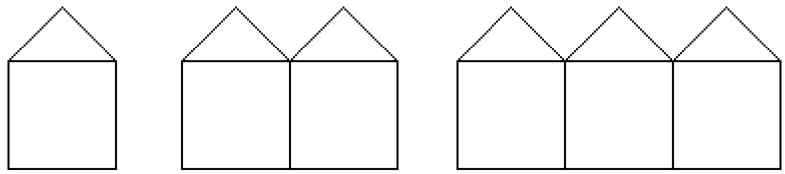
4.

5.

A Few Patterns

Toothpick Houses

Build or draw the following sequence of houses made from toothpicks.



1. Fill in the following table:

Houses	1	2	3	4	7
Number of Toothpicks					

2. See if you can come up with a way to predict the number of toothpicks needed to build any number of houses. Describe your method.

3. Using your method, see if you can fill in the missing parts of the table:

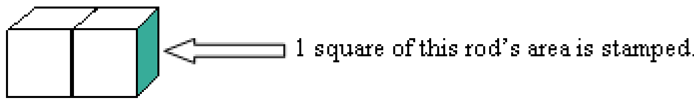
Houses	5	6	8	12	20	100			
Toothpicks							51	76	401

4. How can you calculate the number of houses you can build if you know the number of toothpicks you have? Explain.

5. How would you write your method for question 4 in symbols?

Painting Faces Problem

A company that makes colored rods uses a paint-stamping machine to color the rods. The stamp paints exactly one square of area at a time. Every face of each rod has to be painted, so this length 2 rod would need 10 stamps of paint.



Rod of length 2

1. How many stamps would you need to completely paint rods from lengths 1 to 10? To make this problem easier to think about, you may want to use cubes to build your own rods.

   Use the table below to record your answers. Then look for a pattern.

Length of Rod	Stamps Needed
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

2. Describe any pattern you notice in the table.

3. How would you find the number of stamps needed for a rod of length:
   a. 20?
   b. 56?
   c. 137?
   d. 213?

4. Write a rule you could use to extend the pattern to any length of rod.

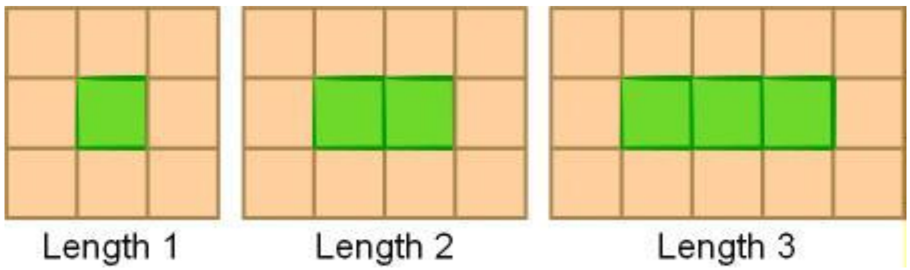
5. If it takes 86 stamps to paint the rod, how long is the rod?

6. If it takes 286 stamps to paint the rod, how long is the rod?

7. Write a rule you could use to find the length of any rod if you know the number of stamps needed to paint it.

Tiling Garden Beds

Gardens are framed with a single row of tiles as illustrated here.
(A garden of length 3 requires 12 border tiles.)



1. Fill in the table to show how many border tiles you would need for these different garden lengths:

Garden Length	1	2	3	6	8
Number of Border Tiles					

2. Use whatever patterns you notice to figure out how many border tiles you would need for a garden of length:
   a. 15
   b. 30
   c. 100

3. Describe a way (in words) to find the number of border tiles for any garden length.

4. Write an expression that represents your method for finding the number of border tiles, given the garden length.

5. Test your method! How long is the garden if the number of border tiles is:
   a. 72 tiles?
   b. 106 tiles?
   c. 432 tiles?

Solutions
Toothpick Houses

1)

Houses	1	2	3	4	7
Number of Toothpicks	**6**	**11**	**16**	**21**	**36**

2) Your answers will certainly vary here. You might notice that:

a) the number of toothpicks increases by 5 each time;

b) the ones digit will always be either 1 (if house number is even) or 6 (if house number is odd); the tens digit will be half that of the house number if the house number is even – if the house number is odd, it will be half of the previous even house number;

c) the number of toothpicks will be 5 x the house number + 1 or **5H + 1**

d) the number of toothpicks will be 6 x the house number – the previous house number (for example: 6 (1) – 0 = 6; 6(2) – 1 = 11; 6(3) – 2 = 16; 6(4) – 3 = 21; etc.
or **6H – (H-1) or 5H + 1.**

3)

Houses	5	6	8	12	20	100	**10**	**15**	**80**
Toothpicks	**26**	**31**	**41**	**61**	**101**	**501**	51	76	401

4) Your answers will vary. See #2 above.

5) See 2c and 2d above. 2b does not easily lend itself to a symbolic expression.

Painting Faces

1)

Length of Rod	Stamps Needed
1	6
2	10
3	14
4	18
5	22
6	26
7	30
8	34
9	38
10	42

2) Answers will certainly vary here. Many people mention right away that the number of stamps needed **increases by 4** each time. You might have also noticed that the number of stamps needed is always **2 more than 4 times the length of the rod.**

WHY? One way to imagine the painting is to think of each cube having 4 faces that will be stamped with paint (top, front side, back side and bottom). The two end cubes will have the exposed right end and the exposed left end painted as well.

If C is the number of cubes, this leads to the rule: 4C + 2.

Another way to think of the pattern is to say that each cube has 6 faces. However, when two cubes are joined, 1 face is covered up on each of them. This leads to the rule: 6 x (the number of cubes) – 2 x (the number of "connections" between cubes. The number of connections between cubes is always one less than the number of cubes.

If C = the number of cubes, you get 6C – 2(C-1) or 4C + 2.

There are various ways to see how the pattern of number connects to the model of the attached cubes. However, whatever way you see the pattern, the expression should simplify to 4C + 2.

3) Using the rule 4C + 2:
a) 82 = 4(20) + 2 b) 226 = 4(56) + 2 c) 550 = 4(137) + 2 d) 854 = 4(213) + 2

4) 4C + 2 or similar stated in words

5) 21: Here, you work the problem backwards. 86 = 4C + 2 and you solve for C. Some students will do so without an equation and will deduct 2 from 86 to get 84, then divide 84 by 4 to get 21.

6) 71: If S = the number of stamps, (S – 2)/4, or first, subtract 2 from the number of stamps, then divide that amount by 4.

Tiling Garden Beds

1) Fill in the table to show how many border tiles you would need for these different garden lengths:

Garden Length	1	2	3	6	8
Number of Border Tiles	8	10	12	18	22

2) a) 36
b. 66
c. 206

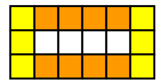
3) Double the length of the garden, then add six more tiles.

Why does this work? One way to look at the pattern is like this:

The rows above and below the garden tiles double the number of tiles in the garden bed.

The two sets of 3 tiles on either end add six.

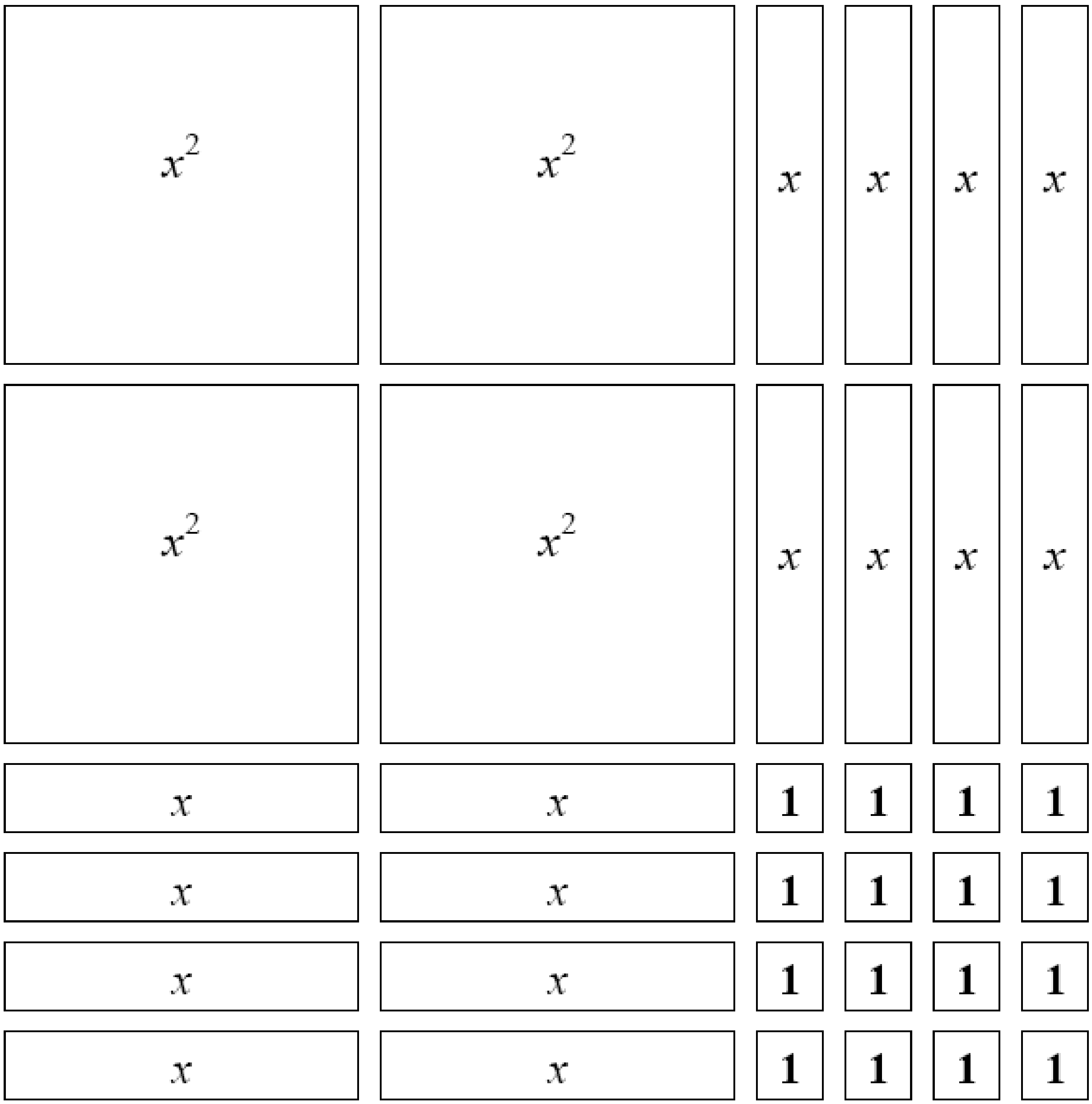
4) If the garden length is L, one expression could be 2L + 6.



5) a) 33
b) 50
c) 213

You can find each of these values by subtracting 6 then dividing by 2.

Algebra Tile Template



Algebra Tile Mat

X

Simon Says

Students need to feel comfortable with algebra tiles in order to use them effectively and learn basic concepts. Introduce the tiles – x2, -x2, x, -x, 1, and -1. Model for students how to "set up" the tiles to show different expressions, as well as defining the basic vocabulary to be used: variable, constant, coefficient, and expression. To ensure that students understand the basics, play "Simon Says." Provide students with sample algebraic expressions and have them use their algebra tiles to show the correct display. Speed up the directions as students become more comfortable using the tiles.

Example:

Simon says show me:

- $2x^2$
- $4x$
- $- x^2$
- 3
- $2x + 3$
- $- x^2 + 4$
- $2x^2 + 6x + 5$
- $-2x^2 - 6x - 5$
- $x^2 - 2x + 3$

You may wish to play a round robin version of "Simon Says" by selecting a student to state an expression with the first person getting it correct providing the next set of directions.

Using Algebra Tiles to Add Like Terms

Use algebra tiles to show each expression and draw your result. Then combine like terms. Show your final answer by circling it.

1.) $3 + x^2$

2.) $x + 3 + 2x$

3.) $3x + 2 + x + 4$

4.) $4x + 1 + 2x + 4$

5.) $4x + 2x - 4 + 5 - x$

6.) $x + 4 + 3x - 3$

7.) $2x^2 + x^2$

8.) $2x^2 + 3x + 2 + 2 x 2$

9.) $x^2 +3 x + 2x2 - x + 2$

Regardless of What You Call the Terms, It's All Just Math!

Translate the following into a mathematical expression.

1. the difference between twice x and y
2. the difference between the square of x and x
3. the quotient of y and 3
4. five times the sum of x and y
5. the sum of 4 times x and y
6. ten less than x
7. the product of a, b, and c
8. the sum of 7 and x
9. x minus 8
10. x less than 8

Try These!

For each of the following, write an expression in terms of the given variable that represents the indicated quantity.

1. The cost of having a mechanic fix your car if he spends *h* hours on the job and charges \$39 for parts and \$45 per hour for labor.

2. The sum of three consecutive even numbers if the first number is *n*.

3. The amount of money in Steve's bank account if he put in *d* dollars the first year, \$600 more the second year than the first year, and twice as much the third year as the second year.

4. The first side of a triangle is *s* yards long. The second side is 3 yards longer than the first side. The third side is three times as long as the second side. What is the perimeter of the triangle in feet?

One Way to Solve (from a fellow instructor)

Hartley opened a food truck business to sell food on the street. On day 2, the business earned \$112. On day 5, the business earned \$367. Hartley assumes that the earnings will continue to increase at the same rate. How much will the business earn on day 10?

S How much will the business earn on day 10?

O On day 2, the business earned \$112.
On day 5, the business earned \$367.
Earnings will continue to increase at the same rate[1].
[1]Not all the facts have to be numeric.

L There is a relationship between two variables. Number of days (*x*) is the independent variable, and money earned (*y*) is the dependent one. Since the rate is constant, there is a linear relationship.
a) Calculate the rate (slope) and the *y*-intercept.
b) Write the linear equation.
c) Solve for the money earned on day 10.

V a) $m = \frac{367-112}{5-2} = \frac{255}{3} = 85$

b = 367 – 85(5) = -58

b) $y = 85x - 58$

c) $y = 85(10) - 58 = 850 - 58 = \$792$

E The amount earned after 10 days is greater than the amount earned after 5 days. This is consistent with what happened from day 2 to day 5. Since the slope and the *y*-intercept are integers, the answer must also be an integer.

A Few Websites to Get You Started!

Free Resources for Educational Excellence. Teaching and learning resources from a variety of federal agencies. This portal provides access to free resources. http://free.ed.gov/index.cfm

PBS Teacher Source. Lesson plans and lots of activities are included in the teacher section of PBS. http://www.pbs.org/teachers

Annenberg Learner. Courses of study in such areas as algebra, geometry, and real-world mathematics. The Annenberg Foundation provides numerous professional development activities or just the opportunity to review information in specific areas of study. http://www.learner.org/index.html

Illuminations. Great lesson plans for all areas of mathematics at all levels from the National Council of Teachers of Mathematics (NCTM. http://illuminations.nctm.org

Inside Mathematics. A professional resource for educators, including classroom examples of innovative teaching methods and insights into student learning. http://insidemathematics.org/index.php/home

Khan Academy. A library of over 2,600 videos covering everything from arithmetic to physics, finance, and history and 211 practice exercises. http://www.khanacademy.org/

The Math Dude. A full video curriculum for the basics of algebra. http://www.montgomeryschoolsmd.org/departments/itv/MathDude/MD_Downloads.shtm

Math Planet. Math Planet is a dedicated web site to the advancement of mathematics. http://library.thinkquest.org/16284/index_s.htm

Geometry Center (University of Minnesota). This site is filled with information and activities for different levels of geometry. http://www.geom.uiuc.edu/

Online Resources for Teaching and Strengthening Fundamental, Quantitative, Mathematical, and Statistical Skills. NICHE. A wide array of resources for the different types of mathematical skills. **http://serc.carleton.edu/NICHE/teaching_materials_qr.html#partone**

National Library of Virtual Manipulatives for Math - All types of virtual manipulatives for use in the classroom from algebra tiles to fraction strips. This is a great site for students who need to see the "why" of math. http://nlvm.usu.edu/en/nav/index.html

Teacher Guide for the TI-30XS MultiView™ Calculator – A guide to assist you in using the new calculator, along with a variety of lesson plans for the classroom. http://education.ti.com/en/us/guidebook/details/en/62522EB25D284112819FDB8A46F90740/30x_mv_tg

http://education.ti.com/calculators/downloads/US/Activities/Search/Subject?s=5022&d=1009

TES. With more than 2.3 million registered online users in over 270 countries and territories, TES provides a wealth of free resources in all academic areas. http://www.tes.co.uk/

Atomic Learning – Tutorials for the TI 30XS MultiView™ Calculator and emulator. http://www.atomiclearning.com/ti30xs

Math in the News. Media4Math. This site provides you with information/articles of how math is used in the real world. http://www.media4math.com/MathInTheNews.asp

Algebra 4 All. A website from Michigan Virtual University with an interactive site for using algebra tiles to solve various types of problems.
http://a4a.learnport.org/page/algebra-tiles

Working with Algebra Tiles. An online workshop that provides the basics of using algebra tiles in the classroom.
http://mathbits.com/MathBits/AlgebraTiles/AlgebraTiles.htm

Teaching Algebra Using Algebra Tiles. An instructor site that provides information on teaching algebra, as well as basic algebraic concepts.
http://www.jamesrahn.com/homepages/algebra_tiles.htm

Key Elements to Algebra Success 46 lessons, homework assignments, and videos.
http://ntnmath.keasmath.com/

Stay in Touch!

- Florida GED® 2014 Preparation Program Frameworks – http://www.fldoe.org/workforce/dwdframe/ad_frame.asp
- GED Testing Service® – www.GEDtestingservice.com
- Twitter at @GEDTesting® – https://twitter.com/gedtesting
- GED® Facebook – https://www.facebook.com/GEDTesting
- YouTube channel – http://www.youtube.com/gedtestingservice
- Common Core State Standards – http://corestandards.org

$-6x^5 + 16x^3 - 11x^2$	$(x - 10)(x + 1)$	$x^2 - 9x - 10$	$(x + 4)(x^2 - 2x + 3)$
$x^3 + 2x^2 - 5x + 12$	$(x + 7)(x + 7)$	$x^2 + 14x + 49$	$(x - 2)(x^2 + 6x - 7)$
$x^3 + 4x^2 - 19x + 14$	$(x + 11)(x - 3)$	$x^2 + 8x - 33$	$(x^2 + 2x - 9)(x - 4)$
$x^3 - 2x^2 - 17x + 36$	$(x - 15)(x - 4)$	$x^2 - 19x + 60$	$(4x^2 - 3x - 2)(x + 12)$
$4x^3 + 45x^2 - 38x - 24$	$(x + 2)(x + 7)$	$x^2 + 9x + 14$	$(-2x)(4x + 7)$
$-8x^2 - 14x$	$(x + 1)(x + 1)$	$x^2 + 2x + 1$	$2x(x^2 + x - 5)$
$2x^3 + 2x^2 - 10x$	$(x + 6)(x - 10)$	$x^2 - 4x - 60$	$-4x^2(3x^2 + 2x - 6)$
$-12x^4 - 8x^3 + 24x^2$	$(x - 15)(x + 4)$	$x^2 - 11x - 60$	$(2x - 5)(-4x)$
$-8x^2 + 20x$	$(x + 2)(x^2 + 3x + 5)$	$x^3 + 5x^2 + 11x + 10$	$3x^2(7x - x^3 - 3)$
$-3x^5 + 21x^3 - 9x^2$	$(x - 5)(x^2 - 2x - 6)$	$x^3 - 7x^2 + 4x + 30$	$(-x)(6x^2 + 5x)$
$-6x^3 - 5x^2$	$(x - 3)(x^2 - 4x - 6)$	$x^3 - 7x^2 + 6x + 18$	$4x^2(3x^3 - 2x^2 - x)$
$12x^5 - 8x^4 - 4x^3$	$(2x + 3)(3x^2 - 4x + 2)$	$6x^3 + x^2 - 8x + 6$	$-x^2(6x^3 - 16x + 11)$

Directions

1. Cut out each of the rectangles above and separate them into two different categories: answers and problems.

2. Turn them face down leaving them in two separate categories.

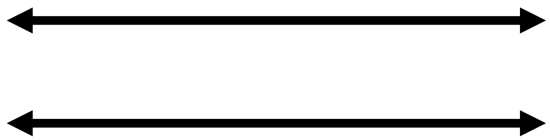
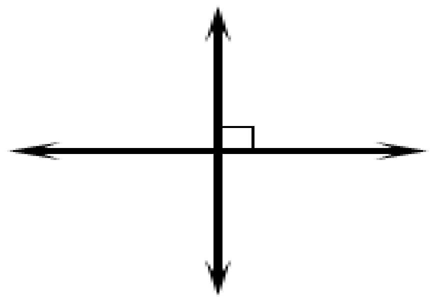
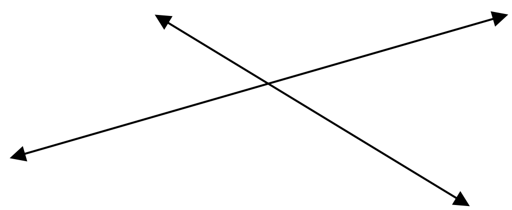
3. One person will turn over 1 problem, but both partners will multiply it out.

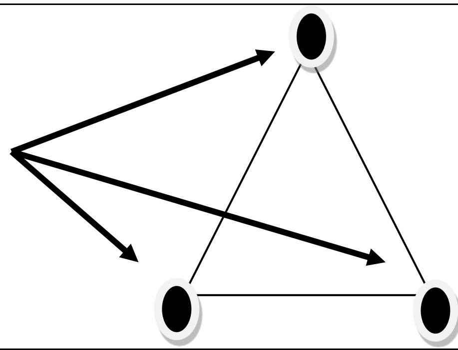
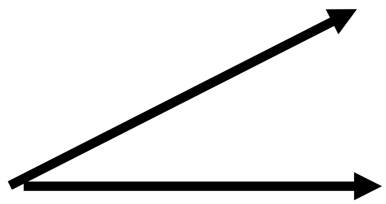
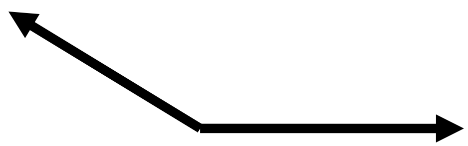
4. After he/she has finished, turn over 1 rectangle from the answer side.

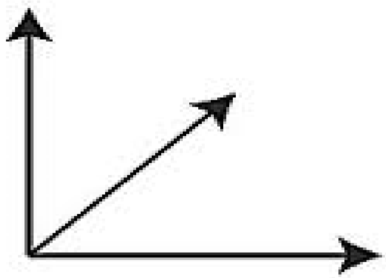
5. If it's a match, keep the pair and choose again.

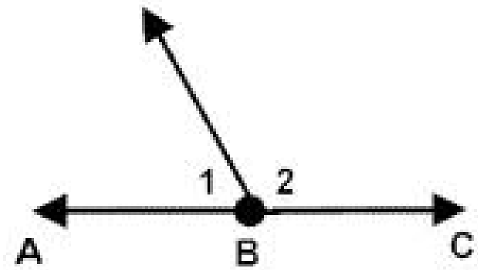
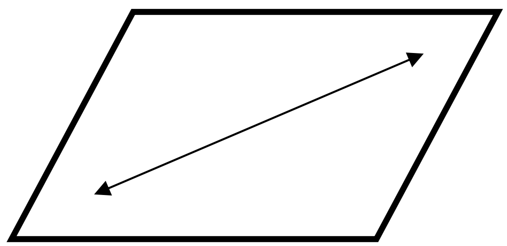
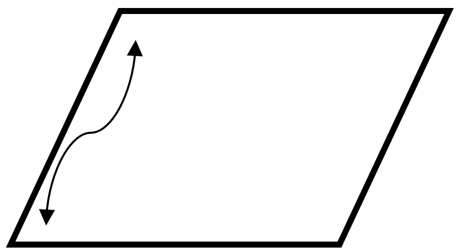
6. If it's not a match, turn them both back over and now the other person chooses.

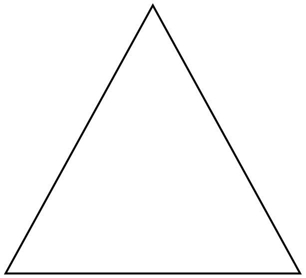
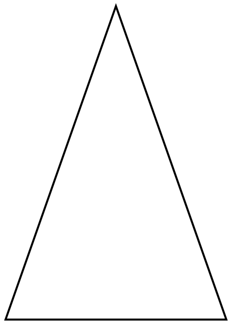
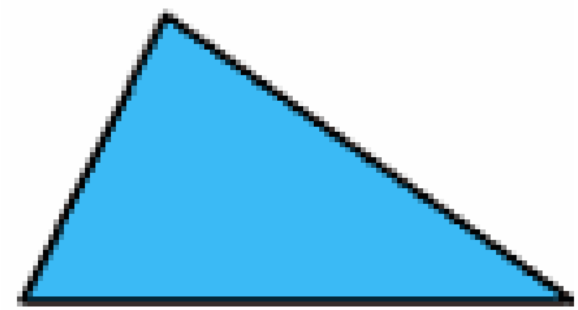
7. The winner is the person with most matches!!!

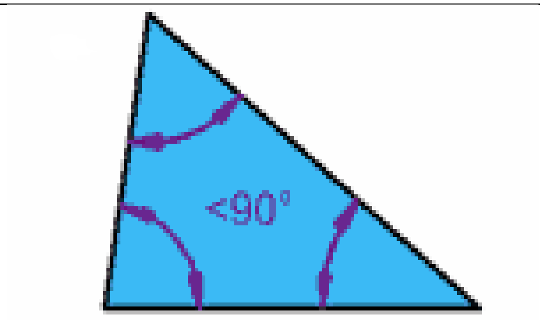
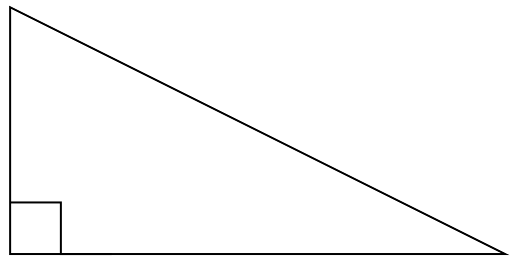
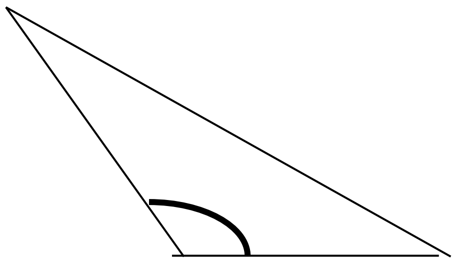
parallel lines		lines that stay same distance from each other forever and never intersect
perpendicular lines		lines that cross at a point and form 90° angles
intersecting lines		lines that cross at a point

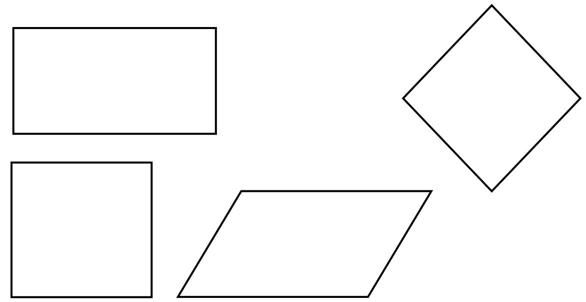
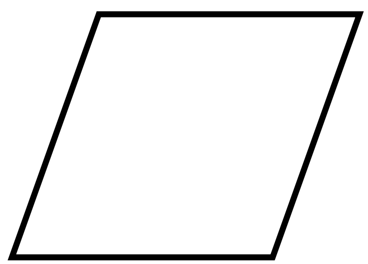
vertices		a point where two or more lines come together; also called a corner on a polygon
acute angle		an angle that is less than 90°
obtuse angle		an angle that is more than 90° and less than 180°

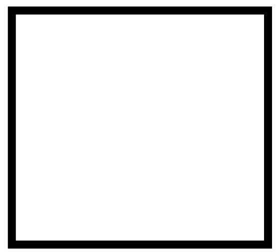
right angle		an angle that is exactly 90°
straight angle		an angle that is exactly 180°
complementary angles		two angles that add up to 90°

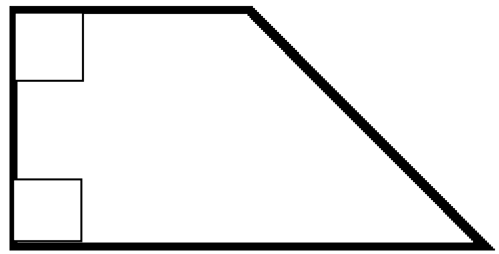
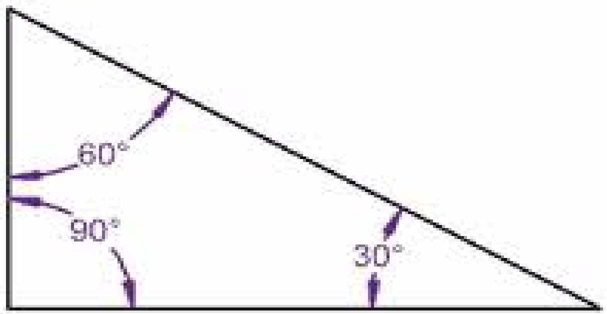
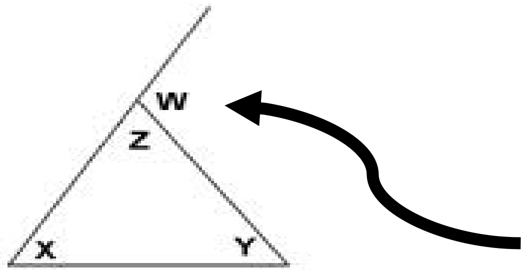
supplementary angles		two angles that add up to 180°
opposite angles		angles that are across from each other; they do not share a side or vertex
adjacent angles		two angles that are formed with a common side

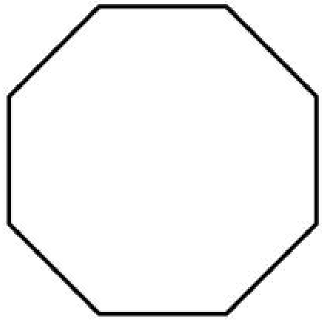
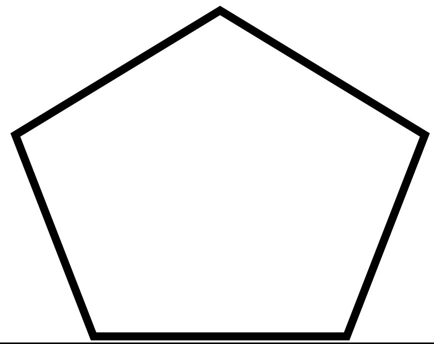
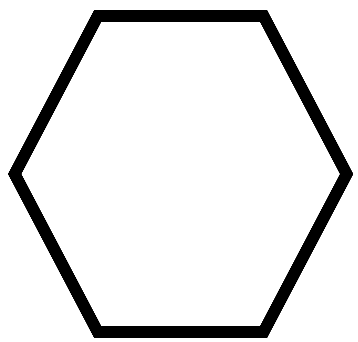
equilateral triangle		a triangle with three equal sides
isosceles triangle		a triangle with two equal sides
scalene triangle		a triangle with no equal sides

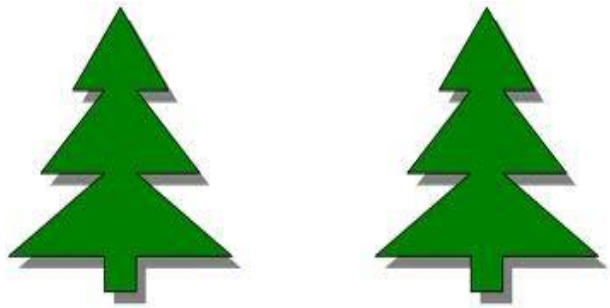
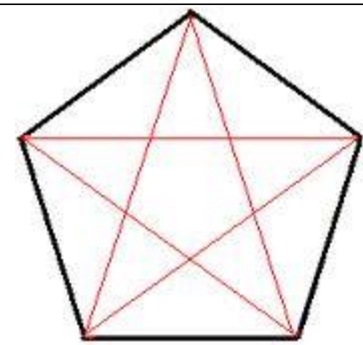
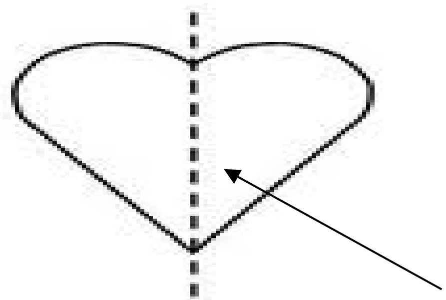
acute triangle		A triangle with all acute angles
right triangle		A triangle with one right angle
obtuse triangle		A triangle with one obtuse angle

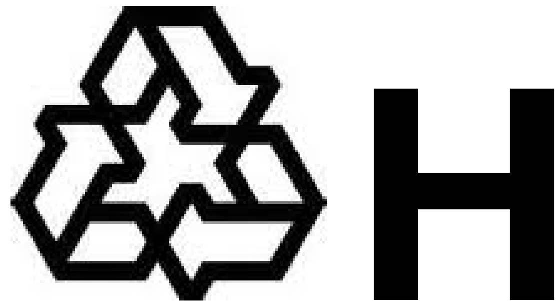
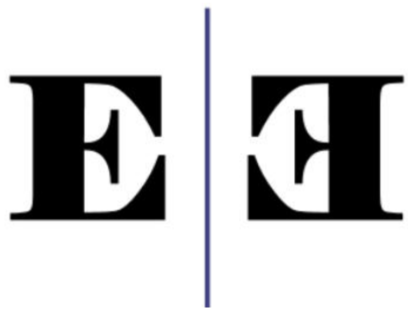
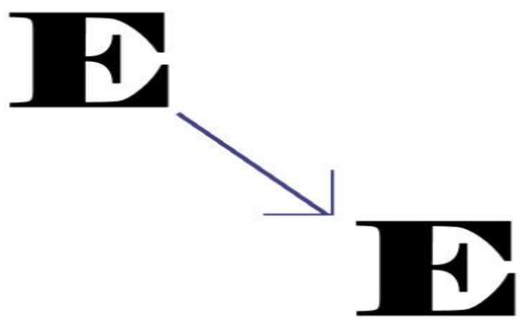
quadrilateral		a four sided polygon; angles total 360°
parallelogram		contains two sets of parallel sides
rhombus		contains two sets of parallel sides that are all congruent

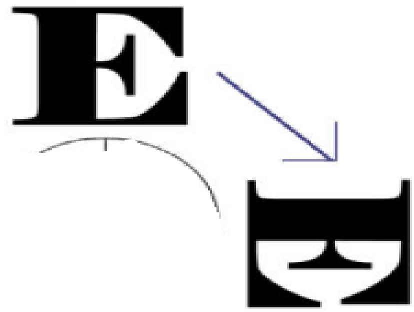
rectangle		contains two sets of parallel sides that form four 90° angles
square		contains two sets of parallel sides that form four 90° angles; all sides are congruent
isosceles trapezoid		a quadrilateral that contains one set of parallel sides; also contains two opposite congruent sides

right trapezoid		a quadrilateral that contains on set of parallel sides and contains two right angles
interior angles		the angles inside of a figure; in a triangle, these add up to 180°
exterior angles		the angles on the outside of a figure when the sides are extended

octagon		an 8-sided polygon
pentagon		a 5-sided polygon
hexagon		a 6-sided polygon

congruent		a word meaning equal or same; it is used to describe figures, sides, and angles
diagonal		a line that cuts across a figure connecting two vertices that are not adjacent
line symmetry		a figure has this when a line can divide it into two congruent parts

rotational symmetry		a figure has this when can be turned around a point and look exactly the same as its original image after some rotating
reflection		a transformation that moves a figure by flipping it across a line
translation		a transformation that moves a figure in a straight line without turning or flipping

rotation		a transformation that moves a figure by turning it