ipdae INSTITUTE FOR THE PROFESSIONAL DEVELOPMENT OF ADULT EDUCATORS

Module: Mathematics

Lesson Title: K-N-W-S: A Problem-Solving Strategy

Standards for the Lesson

Florida Adult Basic Education Mathematics Standards CCR Standards for Mathematical Practice
Make sense of problems and persevere in solving them. (CCR.MP.1)

Interpreting the Standard

1 Standards	2 Skills Included in the Standard	3 Concepts Included in the Standard	4 Through a Particular Context	5 Cognitive Demand/Levels of Thinking	6 Sample Activity
Make sense of problems and persevere in solving them. (CCR.MP.1)	make persevere	sense of problems solving problems	Graphic organizers	DOK 2 DOK 2	Provide students with graphic organizer to use to decode word problems Provide word problems in different areas and achievement levels and have students apply same decoding strategy Have students identify different thinking/problem solving skills

Objectives of the Lesson

Students will:

- Apply a reading strategy to decode word problems
- Identify appropriate problem solving/thinking skills to solve word problems

Materials

- Chart paper/markers
- **Handout A: K-N-W-S**

- Word problems at appropriate level for students in the classroom (Note: Any type of word problem can be used; the importance is to align the type of problem selected with the level of mathematical skills of students in the classroom.)
- **Handout B: Some Helpful Problem-Solving Heuristics** (A resource for identifying different *heuristics* or thinking strategies to use when modeling problem solving for students. This is not meant as a handout for students.)

Instructional Plan

Overview

The standards for Mathematical Practices are important to integrate in math lessons at all achievement levels. This lesson is designed to assist students in applying a problem-solving strategy to all types of word problems. The strategy, adapted from Reading and Writing to Learn in Mathematics: Strategies to Improve Problem Solving by Clare Heidema, can be taught to a multi-level classroom. Differentiate the activity by using different achievement levels of questions.

Process

Introduce the lesson by writing the following on the board: KWL. Ask students if they have ever seen these letters. Students should say that they sometimes use KWL as a strategy for reading, where the K refers to what they know about a topic, the W refers to what they want to learn and the L refers to what they did learn from a lesson. It is a strategy to help students better understand what they are going to read and what they have read.

Share with students that they can use a similar strategy in math to better understand what a word problem is asking. Write the following on the board: K-N-W-S. Have students guess what they think each letter means.

Tell students that when reading word problems, K-N-W-S can help them to determine:

- K – What facts they *KNOW*
- N – What information is *NOT* relevant
- W – *WHAT* the problem wants them to find out
- S – What *STRATEGY* can be used to solve the problem

Share with students that the K-N-W-S strategy helps them to plan, organize, and analyze how to solve word problems. Share with students that effective problem solvers first focus on what needs to be done when they start working with a word problem. The K-N-W-S strategy does this as well. In addition, the K-N-W-S strategy allows students to plan, organize, and analyze how to solve word problems, while teachers can evaluate students' understanding and possible misconceptions about word problems.

Model for students how to use the strategy. Draw a four-column chart on the board or chart paper. Provide each student with a copy of **Handout A: K-N-W-S** or have students construct their own. Using a word problem, model how the columns are used. A sample problem is provided for you to use or you can use a current word problem from a unit that is currently being taught.

Explain how you know which pieces of information belong in each area of the chart.

Next, try another problem using the graphic organizer where students tell you what should be included in each of the sections of the chart. Once students have mastery of how to use the chart, have them work in groups or individually to complete K-N-W-S sheets for other word problems. Have students complete a small number of problems. Remember, it's all about quality, not quantity, when teaching how to decode word problems.

If students need help in identifying different strategies that they can use when solving word problems, refer to **Handout B: Common Strategies for Solving Word Problems**. Use this list as a resource for yourself. Model for students multiple ways to solve problems so that they are aware that there is often more than one way to complete a problem successfully.

You may wish to have students brainstorm some of the strategies that they use in solving word problems, such as:

- Draw a picture
- Make a list
- Guess, check, and revise,
- Find a pattern
- Work backwards
- Write out an equation
- Simplify the problem

Sample Problems

The following are three sample problems that use the graphic organizer to decode the word problems. You may wish to use these problems when modeling the process or problems that you are currently using in the classroom.

Sample Problem (Whole Numbers)

Lena is purchasing apple juice for a party that she is hosting. Lena gives the cashier $30 to pay for 6 liters of apple juice. The cashier gives her $6 in change. Lena is saving $1 per liter. How much does each liter of apple cider cost?

K	N	W	S
$30 is given to the cashier $6 is change	$1 per liter saved	How much does each liter cost?	Guess, check, revise Write an equation to figure out the cost

Sample Problem (Fractions)

Frederic used to pay $0.13 a minute to call his mother long distance. Since he changed to his new phone now pays $0.09 a minute. This month he talked with his mother for 56 minutes. How much will Frederic be charged for this call?

K	N	W	S
$0.09 a minute for phone calls Talked with his mother for 56 minutes	$0.13 per minute cost for old phone company	How much will Frederic have to pay for his phone call to his mother?	Make a chart or table. Write an equation.

Sample Problem (Inequalities)
Video-Online rents movies for $3 each per night. They also offer a MAX Movie plan for $100 per year with two free rentals per month and unlimited rentals at $1 per movie each per night. How many movies must you rent in a year to make the club deal worthwhile?

K	N	W	S
$3 to rent 1 movie Club plan = $100 annual Each movie under plan is $1 2 free movies per month	MAX Movie allows unlimited rentals	How many movies must be rented in a year to make joining the club worthwhile?	Make a chart to compare the costs of both Write an inequality to compare the cost of the regular service to the cost of the club plan

Sample Debriefing Questions

- Why is reading important in math?
- What types of word problems are most difficult for you to complete successfully? Why?
- What are the strategies that you use most often when solving word problems? Why?
- How will using the K-N-W-S strategy help you to better solve word problems?

Modifications for Different Levels

The K-N-W-S strategy for decoding word problems is appropriate for all levels of mathematical understanding. Select word problems that are currently being used with students in order to apply this strategy.

There are many resources on the Internet that can be used in the classroom. One website that provides numerous word problems at different levels is:

- Ohio Mathematics Projects and Resources: http://www.ohiorc.org/for/math/

Handout A: K-N-W-S

K	N	W	S
What facts do I KNOW from the information in the problem?	What information do I NOT need?	What does the problem WANT me to find?	What STRATEGY or operations will I use to solve the problem?

Word Problem:

K	N	W	S

Handout B: Some Helpful Problem-Solving Heuristics[1]

A *heuristic* is a thinking strategy, something that can be used to tease out further information about a problem and thus help you figure out what to do when you don't know what to do. Here are 25 heuristics that can be useful in solving problems. They help you monitor your thought processes, to step back and watch yourself at work, and thus keep your cool in a challenging situation.

Group A

1. Ask somebody else how to do the problem. This strategy is probably the most used world-wide, though it is not one we encourage our students to use, at least not initially.
2. Guess and try (guess, check, and revise). Your first guess might be right! But incorrect guesses can often suggest a direction toward a solution. (A spreadsheet is a powerful aid in guessing and trying. Set up the relationships and plug in a number to see if you get what you want. If you don't, it is easy to try another number. And another.)

Group B

3. Restate the problem using words that make sense to you. One way to do this is to explain the problem to someone else. Often this is all it takes for the light to dawn.
4. Organize information into a table or chart. Having it laid out clearly in front of you frees up your mind for thinking. And perhaps you can use the organized data to generate more information.
5. Draw a picture of the problem. Translate problem information into pictures, diagrams, sketches, graphs, arrows, or some other kind of representation.
6. Make a model of the problem. The model might be a physical or mental model, perhaps using a computer. You might vary the problem information to see whether and how the model may be affected.
7. Look for patterns, any kind of patterns: number patterns, verbal patterns, spatial/visual patterns, patterns in time, patterns in sound. (Some people define mathematics as the science of patterns.)
8. Act out the problem, if it is stated in a narrative form. Acting it out can have the same effect as drawing a picture. What's more, acting out the problem might disclose incorrect assumptions you are making.
9. Invent notation. Name things in the problem (known or unknown) using words or symbols, including relationships between problem components.
10. Write equations. An equation is simply the same thing named two different ways.

Group C

11. Check all possibilities in a systematic way. A table or chart may help you to be systematic.
12. Work backwards from the end condition to the beginning condition. Working backwards is particularly helpful when letting a variable (letter) represent an unknown.
13. Identify subgoals in the problem. Break up the problem into a sequence of smaller problems ("If I knew this, then I could get that").
14. Simplify the problem. Use easier or smaller numbers, or look at extreme cases (e.g., use the minimum or maximum value of one of the varying quantities).

Group D

15. Restate the problem. After working on the problem for a time, back off a bit and put it into your own words in still a different way, since now you know more about it.
16. Change your point of view. Use your imagination to change the way you are looking at the problem. Turn it upside down, or pull it inside out.
17. Check for hidden assumptions you may be making (you might be making the problem harder than it really is). These assumptions are often found by changing the given numbers or conditions and looking to see what happens.
18. Identify needed and given information clearly. You may not need to find everything you think you need to find, for instance.
19. Make up your own technique. It is your mind, after all; use mental actions that make sense to you. The key is to do something that engages you with the problem.
20. Try combinations of the above heuristics.

These heuristics can be readily pointed out to students as they engage problems in the classroom. However, real-world problems are often confronted many times over or on increasingly complex levels. For those kinds of problems, George Polya, the father of modern problem-solving heuristics, identified a fifth class (E) of looking-back heuristics. We include these here for completeness, but also with the teaching caveat that solutions often improve and insights grow deeper after the initial pressure to produce a solution has been resolved. Subsequent considerations of a problem situation are invariably deeper than the first attempt.

Group E

1. Check your solution. Substitute your answer or results back into the problem. Are all of the conditions satisfied?
2. Find another solution. There may be more than one answer. Make sure you have them all.
3. Solve the problem a different way. Your first solution will seldom be the best solution. Now that the pressure is off, you may readily find other ways to solve the problem.
4. Solve a related problem. Steve Brown and Marion Walter in their book, *The Art of Problem Posing*, suggest the "What if not?" technique. What if the train goes at a different speed? What if there are 8 children, instead of 9? What if . . .? Fascinating discoveries can be made in this way, leading to a solution.
5. Generalize the solution. Can you glean from your solution how it can be made to fit a whole class of related situations? Can you prove your result?

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[1] Adapted from Meiring, S. P. (1980). *Problem solving — A basic mathematics goal*. Columbus: Ohio Department of Education.