ipdae INSTITUTE FOR THE PROFESSIONAL DEVELOPMENT OF ADULT EDUCATORS

Module: Mathematical Reasoning

Lesson Title: Speeding Along

Objectives and Standards

Students will:

- Determine whether a relationship is a function
- Calculate the value of a function through a real-world experience

Prerequisite Skills Common Core State Standards	Mathematical Reasoning 2014 GED® Assessment Targets	Mathematical Practices Common Core State Standards
Write and evaluate numerical expressions involving whole-number exponents. (CCSS.Math.Content.6.EE.1) Write, read, and evaluate expressions in which letters stand for numbers. (CCSS.Math.Content.6.EE.2) Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (CCSS.Math.Content.8.F.1)	Represent or identify a function in a formula, a table or graph as having exactly one output (one element in the range) for each input (each element in the domain). (A.7.b) .	Make sense of problems and persevere in solving them. (CCSS.Math.Practice.MP1) Model with mathematics. (CCSS.Math.Practice.MP4) Look for and make use of structure. (CCSS.Math.Practice.MP7)

Materials

- PowerPoint Presentation: Overview of Functions
- **Handout A: Speeding Along – What's My Fine?**
- **Handout B: Speeding Along – What's My Fine? - Answers**
- Calculators

Instructional Plan

Overview

Functions can be a difficult concept for students to understand. This lesson demonstrates how the concept of functions can be connected to real-world situations.

Process

Introduce the lesson by asking the following questions and discussing as a class:

- Have you or someone you know ever received a speeding ticket?
- Have you ever wondered how the ticket amount was calculated?
- Do you think you would be fined the same amount from state to state?

Discuss that fines for speeding are generally based on how fast you're driving over the speed limit. However, speeding fines often differ from state to state. Share with students that in today's lesson, they will learn how to use a mathematical function in order to determine the amount of a speeding ticket in different states.

Write the following words on the board:

- Function
- Input
- Output

Divide the class into small groups. Have each group define the words and share their answers with the class.

Discuss that a function is really any relationship between an input variable and an output variable in which there is exactly one output for each input. Ensure that students understand that a:

> *A function is a mathematical relationship between two variables, where every input variable has one output variable.*

Share with the class that they see functions all the time – some with numbers; some without. The important thing to remember about a function is that there must be exactly one output for each input. Share the following with students to reinforce different types of functional relationships:

> **Input:** the name of a state
> **Output:** that state's capital
>
> **Input:** the side length of a square
> **Output:** the area of that square
>
> **Input:** a word
> **Output:** the first letter of that word
>
> **Input:** radius of a circle
> **Output:** the area of that circle
>
> **Input:** number of hours worked
> **Output:** amount of money earned

Discuss that functions can be important in determining information. For example, suppose you know that Alice makes $100 per day. If you know how many days she works, you can tell me how much money she has made. Her earnings "depend on" how long she works. Her earnings are a function of the amount earned and how long she works.

Provide additional real-world examples of functions to ensure that students understand the concept. The following are examples that you may wish to use.

Example 1:

A car's efficiency in terms of miles per gallon of gasoline is a function. If a car typically gets 20 mpg, and if you input 10 gallons of gasoline, it will be able to travel roughly 200 miles.

Share with students that in this functional representation, they can determine how many miles a car can travel based on the number of gallons used.

Example 2:

A soda, snack, or stamp machine shows an example of a function. The user puts in money, punches a specific button, and a specific item drops into the output slot. (The function rule is the product price. The input is the money combined with the selected button. The output is the product, sometimes delivered along with coins in change, if the user has entered more money than required by the function rule.)

If students need additional review and support of how to determine a function, you may wish to review the slides in the **PowerPoint: Overview of Functions**.

Share with students that in today's lesson, they will be using a function in order to determine the cost of a speeding ticket. Discuss that speeding tickets are determined by how fast a driver is going above the speed limit. The higher the driver's speed is compared to the speed limit, the more the driver will pay. Tell students that to represent this relationship algebraically, a function or equation can be developed.

Model for students the following scenario:

The fine for speeding on the highways of most states is a function of the speed of the car. The speeding fine can be determined by a linear equation; where **F** represents the fine in dollars and **m** represents the number of miles the car is EXCEEDING the posted speed limit.

The speed limit on federal highways is 70 miles per hour (mph). The fine for speeding is $10 per mile above the legal limit plus an additional $40.

To set up the function, I will use F to represent the fine in dollars and m to represent the number of miles the car is exceeding the 70 mph speed limit, so:

F = 10m + 40 where $10 per mile above the speed limit is multiplied by the number of mph above 70 and an additional $40 is added to the amount.

Now, I can calculate for a speeding fine. Let's say that I was traveling 78 mph. By determining that I was traveling 8 mph above the speed limit, I can put that number into the equation and determine my speeding ticket amount.

- F = 10m + 40
- F = 10(8) = $80 + 40
- F = $120

For this calculation, the F (fine in dollars) is $120 and the m (number of miles the car is exceeding the speed limit) is 8.

Ask students if the input could result in any other output. Why or why not? Reinforce that in a function, one input can only have one output. Have students practice using the formula with different inputs.

Provide students with **Handout A: Speeding Along - What's My Fine?** Have students determine the function and complete each table. Share with students that they will be determining the relationship between the miles exceeding the speed limit and the fine.

To debrief the activity, have students share their answers with the class. Have students discuss the relationship between the miles exceeding the speed limit and the fine.

Sample Debriefing Questions

- How does the fine increase or decrease for every 5 miles increased in speed? (Florida's fine increases by $21.25 for every 5 miles; Alabama's fine increases $20.00 for every 5 miles; Georgia's fine increases by $50 for every 5 miles.
- How would you describe in words each state's formula and its relationship between the independent and dependent variable? (Florida - The speeding fine is $4.25 per mile exceeding the speed limit plus an additional $71.50. Vermont - The speeding fine is $4.00 per mile exceeding the speed limit plus an additional $10. Connecticut - The speeding fine is $10 per mile exceeding the speed limit plus an additional $40.)
- How could you graphically represent the different speeding fines for each of the states? (table, graph)
- What are other examples of functional relationships? (various answers)

Assessments/Extensions

1. Have students identify different examples of functional relationships with real world situations.
2. Provide students with different tables and/or graphs and have them identify the input and output, as well as whether the table or graph shows a function. Why or why not.
3. Use different real-world scenarios to have students improve their understanding of functions. The following are examples to get started:
    - Johnny's burger restaurant charges $4.25 for a hamburger. There is an additional fee of $.75 for every topping that is added with a maximum number of 8 toppings. Write a function for amount paid for a hamburger with 0 to 8 toppings.
    - Sarah is saving money for a new car. In 18 months, she would like to have $3,200.00 saved. She currently has $500 and wants to open up a savings account. How much does Sarah need to save each month if she saves the same amount on a monthly basis? Write a function for the given amount saved at any month between 0 and 18 months.
4. Access problems from your math materials or the Internet and have students practice completing the problems. Examples can include problems with money as a function of time; temperature as a function of different types of factors, such as the time of day, the season, etc.; location as a function of time as a person cannot be in two places at the same time.
5. Have students create a function table or graph to visually show a function.

Handout A: Speeding Along – What's My Fine?

1. The fine for speeding on the Florida highway is $4.25 per mile above the legal limit plus an additional $71.50. Write a formula that could be used to calculate the speeding fine for any speed over the legal limit of 70 mph.

2. The fine for speeding on the Alabama highway is $4.00 per mile above the legal limit plus an additional $10.00. Write a formula that could be used to calculate the speeding fine for any speed over the legal limit of 70 mph.

3. The fine for speeding on the Georgia highway is $10.00 per mile above the legal limit plus an additional $40.00. Write a formula that could be used to calculate the speeding fine for any speed over the legal limit of 70 mph.

4. If you were fined for traveling 88 mph, how much would your ticket be in each of the following states? Show your work!

    a. Florida

    b. Alabama

    c. Georgia

5. Complete the following table for each state. Insert the function (formula) that you developed for each state.

Complete the following table. Make sure you insert the formula to determine the function.

Florida: F =

M (miles exceeding)	4.25m + 71.50	F (fine)	(M,F)
5			
10			
15			
20			
25			
30			
35			
40			
45			
50			

Alabama: F =

M (miles exceeding)	4m + 10	F (fine)	(M,F)
5			
10			
15			
20			
25			
30			
35			
40			
45			
50			

Georgia: F =

M (miles exceeding)	4m + 10	F (fine)	(M,F)
5			
10			
15			
20			
25			
30			
35			
40			
45			
50			

Handout B: Speeding Along – What's My Fine? - Answers

1. **F = 4.25m + 71.50**
2. **F = 4m + 10**
3. **F = 10m + 40**
4. Florida – F = 4.25 (18) + 71.50 = $148.00

   Alabama – F = 4 (18) + 10 = $82.00

   Georgia – F = 10 (18) + 40 = $220.00

Florida: F = 4.25m + 71.50

M (miles exceeding)	4.25m + 71.50	F (fine)	(M,F)
5	4.25m + 71.50	$ 92.75	(5, 92.75)
10	4.25m + 71.50	$ 114.00	(10, 114)
15	4.25m + 71.50	$ 135.25	(15, 135.25)
20	4.25m + 71.50	$ 156.50	(20, 156.50)
25	4.25m + 71.50	$ 177.75	(25, 177.75)
30	4.25m + 71.50	$ 199.00	(30, 199)
35	4.25m + 71.50	$ 220.25	(35, 220.25)
40	4.25m + 71.50	$ 241.50	(40, 241.50)
45	4.25m + 71.50	$ 262.75	(45, 262.75)
50	4.25m + 71.50	$ 284.00	(50, 284)

Alabama: F = 4m + 10

M (miles exceeding)	4m + 10	F (fine)	(M,F)
5	4m + 10	$ 30.00	(5, 30)
10	4m + 10	$ 50.00	(10, 50)
15	4m + 10	$ 70.00	(15, 70)
20	4m + 10	$ 90.00	(20, 90)
25	4m + 10	$ 110.00	(25, 110)
30	4m + 10	$ 130.00	(30, 130)
35	4m + 10	$ 150.00	(35, 150)
40	4m + 10	$ 170.00	(40, 170)
45	4m + 10	$ 190.00	(45, 190)
50	4m + 10	$ 210.00	(50, 210)

Georgia: F = 10m + 40

M (miles exceeding)	10m + 40	F (fine)	(M,F)
5	10m + 40	$ 90.00	(5, 90)
10	10m + 40	$ 140.00	(10, 140)
15	10m + 40	$ 190.00	(15, 190)
20	10m + 40	$ 240.00	(20, 240)
25	10m + 40	$ 290.00	(25, 290)
30	10m + 40	$ 340.00	(30, 340)
35	10m + 40	$ 390.00	(35, 390)
40	10m + 40	$ 440.00	(40, 440)
45	10m + 40	$ 490.00	(45, 490)
50	10m + 40	$ 540.00	(50, 540)