Supporting ELLs in Mathematics

CCSS-aligned Mathematics Tasks with Annotations and Other Resources

Understanding Language | Language, Literacy, and Learning in the Content Areas

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Supporting ELLs in Mathematics:

Mathematics Tasks with Annotations and Other Resources for Implementing the Common Core State Standards

Developed for the Understanding Language Initiative

PREFACE[1]

Judit Moschkovich

The goal of these materials is to illustrate how mathematics tasks that are aligned with the Common Core State Standards (CCSS) for Mathematics can be used to support mathematics instruction and the learning of English Language Learners (ELLs), at three grade spans (elementary, middle, and high school). We used or adapted tasks from two publicly accessible curriculum projects, Inside Mathematics and Mathematics Assessment Project.

The resources provided here are based on the premise that ELLs develop mathematical proficiency as well as the linguistic resources to express that proficiency by actively participating in mathematical practices and rigorous mathematical reasoning that is well scaffolded by instruction. The eight Common Core Standards for Mathematical Practice focus on key aspects of mathematical expertise. These eight standards set expectations for students to be engaged in apprenticeships in mathematical activities that, over time, simultaneously build their procedural fluency, conceptual understanding, and participation in mathematical reasoning and sense making (Moschkovich, 2012). The additional support offered to ELLs through the UL resources is intended to scaffold their participation in these activities.

These materials have been reviewed by the Understanding Language Initiative's Mathematics Work Group and by a group of expert reviewers (see pages 23-25). All materials will be online at the Understanding Language website: http://ell.stanford.edu.

These materials and recommendations for teaching practice are based on research findings that often run counter to commonsense notions of *language*. There are multiple uses of the terms *language*, *academic language*, or *the language of mathematics*. Many interpretations of these terms for teaching practice reduce the meaning of academic language in mathematics to single words and the proper use of grammar. In contrast, these materials use a more complex view of mathematical language as not only specialized vocabulary but also as extended discourse that includes syntax,

[1] This preface is an adapted version of the introduction to the Understanding Language ELA Unit.

organization, the mathematics register (Halliday, 1978), and discourse practices (Moschkovich, 2007).

From the perspective of the Understanding Language initiative, language is an activity. In these resources, "the language of mathematics" does not mean a list of vocabulary or technical words with precise meanings but the communicative competence necessary and sufficient for competent participation in mathematical discourse practices (Moschkovich, 2012).

Although learning vocabulary may be necessary, it is not sufficient. Learning to communicate mathematically and participate in mathematical discussions is not simply a matter of learning vocabulary. During discussions in mathematics classrooms, students are learning to describe relationships, make generalizations, and use representations to support their claims. The question is not whether students who are ELLs should learn vocabulary but rather how instruction can best support students to learn vocabulary *as they actively engage in mathematical reasoning about important mathematical topics*. Therefore, these materials and recommendations stress the importance of creating (and supporting students) in engaging in rich mathematical discussions.

The annotated tasks provided here all require that teachers develop skills and strategies for leading, supporting, and orchestrating mathematical discussions, whether these occur in small groups or with the whole class. These strategies are best learned in the context of a particular mathematical topic—for example, learning what the best questions are to support algebraic thinking (Driscoll, 1999), or geometric thinking (Driscoll, 2007). These strategies are also best learned through long-term professional development that engages teachers in observation, watching video, sharing lessons, etc. These skills for teaching mathematics are fundamental to supporting students in achieving the expectations set by the CCSS and are essential for supporting ELLs. Therefore, in Appendix A, we provide pointers to materials (books, videos, etc.) that can be used to support teachers in learning to orchestrate mathematical discussions.

REFERENCES FOR PREFACE

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers, grades 6–10.* Portsmouth, NH: Heinemann.

Driscoll, M. J., DiMatteo, R. W., Nikula, J., & Egan, M. (2007). *Fostering geometric thinking: A guide for teachers, grades 5–10*. Portsmouth, NH: Heinemann.

Halliday, M. A. K. (1978). Sociolinguistics aspects of mathematical education. In M. Halliday (Ed.), *The social interpretation of language and meaning* (pp. 194–204). London: University Park Press.

Moschkovich, J. N. (2007). Examining mathematical discourse practices. *For The Learning of Mathematics, 27*(1), 24–30.

Moschkovich, J. N. (Ed.) (2010). *Language and mathematics education: Multiple perspectives and directions for research*. Charlotte, NC: Information Age Publishing.

Moschkovich, J. N. (2012). Mathematics, the Common Core, and language: Recommendations for mathematics instruction for ELs aligned with the Common Core. *Proceedings of the "Understanding Language" Conference*. Stanford, CA: Stanford University. Available online at http://ell.stanford.edu.

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WHAT HAVE WE DONE, HOW, AND WHY?

We began with mathematics tasks developed by the Mathematics Assessment Resource Service's Mathematics Assessment Project, and Inside Mathematics.

The goal was not to reduce language demands by altering the mathematics tasks but instead to provide support and scaffolding for ELLs to learn how to manage complex text in mathematics. There are several reasons not to adapt the texts of a task:

- Changing the language of a task can change the mathematical sense of the task.
- It is not yet clear which adaptations are best to make for which students, for which purposes, or at which times.
- Instruction should support students in understanding complex mathematical texts because they are likely to appear in curriculum and assessment materials.
- Experiences that allow ELLs to engage (with support) with authentic language used in mathematics can provide opportunities for their continued language development.

We then produced three types of resources to support teachers in learning to use CCSS-aligned mathematics tasks with ELLs:

1. Mathematics Tasks with Annotations, describing how to use a CCSS-aligned mathematics task with ELLs. (These are available as separate documents on the Understanding Language website.)

2. Pointers to Professional Development Materials (books, videos, etc.) that can be used by teachers to learn to orchestrate mathematical discussions. (This is Appendix A of this document.)

3. Templates for Language of Mathematics Tasks provides templates for five language-focused tasks. Teachers can use these templates to design and write their own Language of Mathematics tasks to fit a mathematics task of their choice. (This is Appendix D of this document.)

These resources were developed using the "Key principles for mathematics instruction for ELLs" (pages 9-15 of this document) and the "Guidelines for design of mathematics instructional materials for ELLS" (pages 16-22 of this document).

Descriptions of Resources

There are three types of resources:

1. **Mathematics Tasks with Annotations**
2. **Pointers to Professional Development Materials**
3. **Language of Mathematics Task Templates**

These resources are provided for teachers as exemplars of the type of instruction that supports mathematical reasoning and sense making using complex, rigorous, and academically challenging tasks or lessons for all students, including ELLs. Teachers are encouraged to generate their own lessons using these resources.

1. Mathematics Tasks with Annotations (on UL website)

Elementary School | *Roger's Rabbits*

Middle School | *Making Matchsticks*

High School | *Sidewalk Patterns*

High School | *Creating Equations*

Each mathematics task presents opportunities for students to develop skills called for by the Standards for Mathematical Practice. In addition, each task provides grade-level–appropriate opportunities for students to comprehend and produce mathematical language. They have a variety of structures: some are scaffolded, allowing students to familiarize themselves with a problem situation before being asked to perform tasks of higher cognitive demand, while one task presents students with an open-ended problem and asks them to collaborate in order to reach a reasonable consensus solution.

The annotations describe the standards afforded by each task. Each annotation includes comments and suggestions for how to use the task with ELLs, as well as “Language of Mathematics” tasks designed to support ELLs. For each task, annotations provide the following information:

- Core mathematical ideas in the task.
- CCSS for Mathematical Content, CCSS for Mathematical Practice, and CCSS for ELA/Literacy.
- Comments on the pedagogical purposes of the task.
- Suggestions for using the task with ELLs.
- Language of Mathematics tasks (one or more) with teacher directions and student materials.

2. Pointers to Professional Development Materials (Appendix A)

A central skill in teaching ELLs mathematics is supporting mathematical discussions in the classroom. This aspect of teaching mathematics is fundamental to teaching mathematics for understanding, supporting students in the CCSS, and engaging students in the mathematical practices. It is also essential for supporting ELLs to develop both mathematical proficiency and language. There are resources already available that can support teachers in developing these skills. Therefore, we provide Pointers to Professional Development Materials (books, videos, etc.) that can be used by teachers to learn to orchestrate mathematical discussions. Although teachers can read the materials on their own, the best settings for this type of professional development would be long-term study groups or professional development experiences.

3. Language of Mathematics Task Templates (Appendix D)

These templates are general descriptions for five language focused activities. Teachers can use these templates to write their own Language of Mathematics Tasks to fit a mathematics task of their choice.

The Language of Mathematics tasks focus on two issues, reading mathematics problems and using vocabulary, not because we think that these are the most central for learning mathematics *or* language but, instead, for several pragmatic reasons: a) These two issues are often raised by practitioners as the major stumbling blocks they face when teaching ELLs, and b) There were exemplars of tasks addressing these two issues which had been already piloted used with teachers. Descriptions for these activities and student materials were either: adapted from the Understanding Language, English Language Arts Unit, adapted from materials already used by teacher professional development professionals (R. Santa Cruz and H. Asturias), or recommended as "good bets" by researchers with expertise in how to scaffold vocabulary and support reading comprehension. The materials draw, in large part, on papers prepared for the Spring 2012 Understanding Language Conference at Stanford University (http://ell.stanford.edu/papers/practice) and the ELA unit written by Walqui, Koelsch, & Schmida (http://ell.stanford.edu/teaching_resources/ela).

Principles for Mathematics Instruction for ELLs

Principles for Mathematics Instruction for ELLs

Judit Moschkovich

In the era of the Common Core, the need for research-based principles for ELL instruction in mathematics cannot be overstated. The first section of this document describes principles derived from research on instruction for ELLs, from instruction in mathematics, and from characteristics of instruction aligned with the Common Core. The second section describes principles derived from research specific to language and mathematics education. Together, the four types of principles provide research-based guidance for teaching mathematics to ELLs.

1. What is effective instruction for ELLs?

Although it is difficult to make generalizations about the instructional needs of all students who are learning English, instruction should be informed by knowledge of students' experiences with instruction, language history, and educational background (Moschkovich, 2010). In addition, research suggests that high-quality instruction for ELLs that supports student achievement has two general characteristics: a view of language as a resource rather than a deficiency, and an emphasis on academic achievement, not only on learning English (Gándara & Contreras, 2009).

Overall, students who are labeled as ELLs are from non-dominant communities and they need access to curricula, teachers and instructional techniques proven to be effective in supporting the academic success of these students. The general characteristics of such environments are that curricula provide "abundant and diverse opportunities for speaking, listening, reading, and writing" and that instruction should "encourage students to take risks, construct meaning, and seek reinterpretations of knowledge within compatible social contexts" (Garcia & Gonzalez, 1995, p. 424).

2. What is effective mathematics instruction?

According to a review of the research (Hiebert & Grouws, 2007), mathematics teaching that impacts student achievement and promotes conceptual development has two central features. First, teachers and students attend explicitly to concepts. Second, teachers give students time to wrestle with important mathematics. Another research-based recommendation is to maintain tasks at high cognitive demand, for example, by encouraging students to explain their problem-solving and reasoning (AERA 2006; Stein, Grover, & Henningsen, 1996). Mathematics instruction for ELLs should follow these general recommendations for effective mathematics instruction to focus on mathematical concepts, emphasize the connections among those concepts, use high cognitive demand mathematical tasks, and maintain high cognitive demand throughout lessons.

3. What is mathematics instruction that is aligned with the CCSS?

First and foremost, mathematics instruction that is aligned with the CCSS means *teaching mathematics for understanding*. Students should use and explain connections between representations, share and refine their reasoning, and develop meaning for symbols.

Mathematics instruction for ELLs should align with the CCSS, particularly in these four ways:

1. *Balance conceptual understanding and procedural fluency.* Instruction should balance student activities that address important conceptual and procedural knowledge and connect the two types of knowledge.
2. *Maintain high cognitive demand.* Instruction should use high cognitive demand mathematics tasks and maintain the rigor of tasks throughout lessons and units.
3. *Develop productive beliefs.* Instruction should support students in developing beliefs that mathematics is sensible, worthwhile, and doable.
4. *Engage students in mathematical practices*. Instruction should provide opportunities for students to develop the kind of expertise described in the Common Core State Standards for mathematical practice.
    - Make sense of problems and persevere in solving them.
    - Reason abstractly and quantitatively.
    - Construct viable arguments and critique the reasoning of others.
    - Model with mathematics.
    - Use appropriate tools strategically.
    - Attend to precision.
    - Look for and make use of structure.
    - Look for and express regularity in repeated reasoning.

    To help students acquire such expertise, instruction should provide opportunities for students to solve problems, model with mathematics, identify and explain connections between different representations, communicate their thinking, and construct and critique arguments.

Connecting Mathematical Content to Language

Mathematics instruction for ELLs should follow the three groups of principles described in the previous section. In addition, there are several recommendations that are specific to mathematics instruction for ELLs.

Research shows that ELLs, even as they are learning English, can participate in discussions where they grapple with important mathematical content. Instruction for this population should not emphasize low-level language skills over opportunities to actively communicate about mathematical ideas. Research on language and mathematics education provides three general guidelines for instructional practices for teaching ELLs mathematics (Moschkovich, 2010). Mathematics instruction for ELLs should address much more than vocabulary and support ELLs' participation in mathematical discussions as they learn English. Instruction should also draw on multiple resources available in classrooms (objects, drawings, graphs, and gestures) as well as home languages and experiences outside of school. These general guidelines are expanded as the following instructional principles. (For more detailed versions of these principles see Moschkovich, 2012.)

Principle 1. *Focus on students' mathematical reasoning, not accuracy in using language.*

- Instruction should focus on uncovering, hearing, and supporting students' mathematical reasoning, not on accuracy in using language (Moschkovich, 2010).
- Recognize students' emerging mathematical reasoning.
- Focus on the mathematical meanings learners construct, not the mistakes they make or the obstacles they face (Moschkovich, 2007b).

Principle 2. *Focus on mathematical practices, not language as single words or definitions.*

- Instruction should move away from simplified views of language and interpreting "language" as vocabulary, single words, grammar, or a list of definitions (Moschkovich, 2007a, 2010).
- An overemphasis on correct vocabulary and formal language limits the linguistic resources teachers and students can use to learn mathematics with understanding.

- Instruction should provide opportunities for students to actively use mathematical language to communicate about mathematical situations.
- Instruction should provide opportunities for students to actively engage in mathematical practices such as reasoning, constructing arguments, looking for and expressing structure and regularity, etc.

Principle 3. *Recognize the complexity of language in mathematics classrooms and support students in engaging in this complexity.*

Language in mathematics classrooms includes multiple:

- Representations (objects, pictures, words, symbols, tables, graphs).
- Modes (oral, written, receptive, expressive).
- Kinds of written texts (textbooks, word problems, student explanations, teacher explanations).
- Kinds of talk (exploratory and expository).
- Audiences (presentations to teacher, to peers, by teacher, by peers).

Principle 4. *Treat everyday and home languages as resources, not obstacles.*

- Everyday language and academic language are interdependent and related—not mutually exclusive (Moschkovich, 2010).
- Everyday language and experiences are not necessarily obstacles to developing academic ways of communicating in mathematics (Moschkovich 2007a, 2007b).
- Home languages provide resources for mathematical reasoning and communication (Moschkovich 2007b, 2007c, 2009, 2011).

References for Principles

American Educational Research Association (AERA). (2006). Do the math: Cognitive demand makes a difference. *Research Points 4*(2). Retrieved from http://www.aera.net/Publications/ResearchPoints/tabid/10234/Default.aspx

Gándara, P., & Contreras, F. (2009). *The Latino education crisis: The consequences of failed social policies.* Cambridge, MA: Harvard University Press.

Garcia, E., & Gonzalez, R. (1995). Issues in systemic reform for culturally and linguistically diverse students. *Teachers College Record, 96*(3), 418–431.

Hiebert, J., & Grouws, D. (2007). Effective teaching for the development of skill and conceptual understanding of number: What is most effective? Retrieved from http://www.nctm.org/clipsandbriefs.aspx (Based on: Hiebert, J., & Grouws, D. (2007). The effects of classroom mathematics teaching on students' learning. *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Reston, VA: National Council of Teachers of Mathematics.)

Moschkovich, J. N. (2007a). Beyond words to mathematical content: Assessing English learners in the mathematics classroom. In A. H. Schoenfeld (Ed.), *Assessing mathematical proficiency* (pp. 345–352). New York: Cambridge University Press.

Moschkovich, J. N. (2007b). Bilingual mathematics learners: How views of language, bilingual learners, and mathematical communication impact instruction. In N. Nasir & P. Cobb (Eds.), *Diversity, equity, and access to mathematical ideas* (pp. 89–104). New York: Teachers College Press.

Moschkovich, J. N. (2007c). Using two languages while learning mathematics. *Educational Studies in Mathematics, 64*(2), 121-144.)

Moschkovich, J. N. (2009). *Using two languages when learning mathematics: How can research help us understand mathematics learners who use two languages?* Research Brief and Clip, National Council of Teachers of Mathematics, retrieved from http://www.nctm.org/uploadedFiles/Research_News_and_Advocacy/Research/Clips_and_Briefs/Research_brief_12_Using_2.pdf

Moschkovich, J. N. (Ed.) (2010). *Language and mathematics education: Multiple perspectives and directions for research.* Charlotte, NC: Information Age Publishing.

Moschkovich, J. N. (2011). Supporting mathematical reasoning and sense making for English Learners. In M. Strutchens & J. Quander (Eds.), *Focus in high school mathematics: Fostering reasoning and sense making for all students* (pp. 17–36). Reston, VA: NCTM.

Moschkovich, J.N. (2012). Mathematics, the Common Core, and Language: Recommendations for mathematics instruction for ELs aligned with the Common Core. *Proceedings of the Understanding Language Conference, 2012.* Stanford University, CA. Retrieved from http://ell.stanford.edu/publication/mathematics-common-core-and-language

Stein, M.K., Grover, B., & Hennigsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. American Educational Research Journal, 3 (2), 455-488.

Guidelines for Design of Mathematics Instruction and Materials for ELLs

Guidelines for Design of Mathematics Instruction and Materials for ELLs

Judit Moschkovich

in collaboration with the Understanding Language Initiative's Mathematics Work Group members Phil Daro and Tina Cheuk. These Guidelines *are based on the* ELA Guidelines *written by the Understanding Language ELA Work Group: George Bunch (chair), Martha Inez Castellon, Susan Pimentel, Lydia Stack, and Aída Walqui.*

One aim of the Understanding Language (UL) Mathematics Work Group is to provide general guidelines for instructional design that maximize alignment with the Common Core mathematics standards for ELLs. Work in the service of this goal has informed and been informed by UL's *Key Principles for ELL Instruction.* The *Key Principles* are meant to guide educators and administrators as they work to help ELLs meet standards in various content areas.

The purpose of the *Mathematics Guidelines* is to move toward a shared framing of approaches to designing mathematics instruction and materials for ELLs in ways aligned with the Common Core State Standards.

These guidelines draw in part on papers prepared for the Spring 2012 Understanding Language Conference at Stanford University (http://ell.stanford.edu/papers/practice), and were modeled after the *Guidelines for ELA Instructional Materials Development*. In fact, some guidelines are taken verbatim from the *ELA Guidelines* or modified only by referring to mathematics instead of ELA.

Although developed to be consistent with the UL project-wide *Key Principles* and to parallel the *ELA Guidelines,* the *Mathematics Guidelines* are distinct in that they specifically address the Common Core State Standards for Mathematics and are intended to directly inform the selection, adaptation, or use of mathematics instructional materials to address the needs of ELLs.[2]

[2] Neither these *Guidelines* nor the UL *Principles* should be confused with the *Publisher's Criteria for the Common Core State Standards in Mathematics*, a much more extensive document intended for commercial textbook companies and curriculum developers that was prepared by the CCSSO and others independent from the work of Understanding Language and which does not focus explicitly on ELLs.

GUIDELINES FOR MATHEMATICS INSTRUCTIONAL MATERIALS DEVELOPMENT

1. **Focus on the Standards for Mathematical Practice.** Consider how the Standards for Mathematical Practice (SMP) are addressed across the various modes of communication (reading, writing, listening, speaking) that will be used during instruction.

    1. Make sense of problems and persevere in solving them.
    2. Reason abstractly and quantitatively.
    3. Construct viable arguments and critique the reasoning of others.
    4. Model with mathematics.
    5. Use appropriate tools strategically.
    6. Attend to precision.
    7. Look for and make use of structure.
    8. Look for and express regularity in repeated reasoning.

    When considering SMP 6 (Attend to precision) during instruction for ELLs it is important to remember that emerging language may sometimes be imperfect. It is also crucial to recognize that mathematical precision does not lie in using one precise individual word, but in making precise mathematical claims. Lastly, mathematically precise statements need not be expressed in full sentences.

2. **Keep tasks focused on high cognitive demand, conceptual understanding, and correspondences among representations.** Mathematics instruction for ELLs should follow the general recommendations for high-quality mathematics instruction: a) Focus on mathematical concepts and the connections among those concepts; and b) Use and maintain high-cognitive-demand mathematical tasks, for example, by encouraging students to explain their problem solving and reasoning (AERA, 2006; Stein, Grover, & Henningsen, 1996). Explanations and justifications need not always include words. Instruction should support students in learning to develop oral and written explanations, but students can also show conceptual understanding by using diagrams and other representations. For example, students might use an area model to *show* that two fractions are equivalent or how multiplication by a positive fraction smaller than one makes the result smaller.

3. **Create multiple instructional pathways that provide students with different academic and linguistic backgrounds access to, engagement with, and achievement of the standards.** The goal is to create structures that allow students to participate in

classroom communities that serve as "apprenticeship" opportunities that lead, over time, to students' acquisition of the expertise described in the Standards for Mathematical Practice.

4. **Facilitate students' production of different kinds of reasoning (algebraic, geometric, statistical, etc.) and comparisons of reasoning.** Include different language functions such as describing, comparing, and arguing. Although sentence frames can be useful scaffolds, these should be used flexibly and fluidly, more as *sentence starters* than rigid formulas for producing perfect sentences.
5. **Facilitate students' participation in different kinds of participant structures—from informal, collaborative group interactions to formal presentations—in ways that allow them to use their own existing linguistic resources and collaborate with others to articulate ideas, interpret information, and present and defend claims.** Support student participation in classroom mathematical discussions in multiple settings—pairs, small groups, and whole-class discussions. Consider the spectrum of ELLs both in terms of English proficiencies, mathematical proficiencies, and literacy in their first language. For examples of different types of support for ELLs at different language proficiency levels, see *Multiple Pathways for ELLs at Different Levels* (Appendix C, adapted from the ELA unit by Walqui, Koelsch, & Schmida, 2012).
6. **Focus on language as a resource for reasoning, sense making, and communicating with different audiences for different purposes.** Activities calling students' attention to features of language (e.g., grammatical structures, vocabulary, and conventions of written and oral language) should only occur in conjunction with, and in the service of, engagement with the text and representations of a mathematical task, and ideas and practices relevant to its solution. There are many ways to address vocabulary that include introducing, using, and reviewing. Vocabulary need not be pre-taught or introduced in isolation but instead should be included in activities that involve high cognitive demand mathematical work: reasoning, sense making, explaining, comparing solutions, etc. To introduce new vocabulary, it is useful for students to first have a successful and engaging experience discussing their mathematical reasoning and developing their conceptual understanding, then label, discuss, and review the vocabulary, grounding meanings in the students' mathematical work.
7. **Prepare students to deal with typical texts in mathematics, both in word problems and mathematics textbooks.** Typical texts in mathematics include written texts such as word problems, assessment problems, textbook explanations, and scenarios for

modeling. Oral texts include explanations, descriptions of solutions, conjectures, justifications, etc. There are several reasons not to adapt the language of a task:

- Changing the language of a task can change the mathematical sense of the task.
- It is not yet clear which adaptations are best to make for which students, for which purposes, or at which times.
- Instruction should support students in understanding complex mathematical texts as they are likely to appear in curriculum and assessment materials.
- Experiences that allow ELLs to engage (with support) with authentic language used in mathematics can provide opportunities for their continued language development.

Thus, the goal of instruction should not necessarily or always be to “reduce language demands” but instead to provide support and scaffolding for ELLs to learn how to manage complex text in mathematics.

8. **Consider how extended instructional units provide students with opportunities to encounter and engage with various kinds of text complexity.** For example, consider mathematical texts with multiple levels of meanings and purposes: textbook explanations, word problems, written/oral explanations produced by the teacher and other students, etc. Texts should include the complexity that is expected to be encountered in typical mathematics tasks used in assessments. Such tasks may include language that is ambiguous for some ELLs, e.g., “table” may mean a classroom table or a mathematical table. Language may be “inconsiderate” in its various levels of semantic and syntactic complexity and requirements for background knowledge (see Tables 1-2-3 in Section B in CCSSO’s *Framework for English Language Proficiency Development*). Note that not *all* texts need to represent *all* types of complexity (see the next guideline).

9. **Prioritize particular aspects of mathematical text complexity for pedagogical focus at different points during instructional units, providing necessary levels of support for students to engage with those areas of complexity.** For ELLs (and other inexperienced or struggling readers), supports for non-targeted areas of text complexity can provide opportunities for students to focus on what is being prioritized. For example, if the pedagogical focus is on the ambiguous or inconsiderate language used in mathematics word problems or textbooks, then shorter or excerpted texts might be selected, and students might be provided with short notes on the meaning of key words or phrases encountered in the text. If the focus is on another aspect

of text complexity (for example, representing a word problem as an expression or equation or producing a diagram or graph for a problem presented in text) then ambiguous or inconsiderate language might be annotated for students. Different texts might be chosen to emphasize different aspects of text complexity, or a single text might be read multiple times, with a different focus for each reading.

10. **Provide opportunities to activate and build students' background knowledge in ways that do not foreclose opportunities for them to engage with typical mathematical texts.** Leveraging students' existing background knowledge to build new knowledge can occur in a number of ways before and during a task, lesson, or unit—without preempting the text, translating its contents for students, telling students what they are going to learn in advance of reading a particular text, pre-teaching vocabulary, or "simplifying" the text itself.

11. **Recognize that all students, including ELLs, have linguistic resources that can be employed to engage with activities designed to meet the Common Core State Standards that include typical mathematical texts.** As they continue to expand their linguistic repertoires in English, students can use a wide variety of linguistic resources—including home languages, everyday language, developing proficiency in English, and nonstandard varieties of English—to engage deeply with the kinds of instruction called for in the preceding guidelines (Bunch, Kibler, & Pimentel, 2012).

References for Guidelines

American Educational Research Association (AERA). (2006). Do the math: Cognitive demand makes a difference. *Research Points, 4*(2).

Bunch, G., Kibler, A., & Pimentel, S. (2012). *Realizing opportunities for English learners in the Common Core English Language Arts and Disciplinary Literacy Standards*. Paper presented at the Understanding Language Conference, Stanford, CA. Available online http://ell.stanford.edu/

Council of Chief State School Officers. (2012). *Framework for English Language Proficiency Development Standards corresponding to the Common Core State Standards and the Next Generation Science Standards.* Available online http://www.ccsso.org/Resources/Publications/The_Common_Core_and_English_Language_Learners.html

Hiebert, J., & Grouws, D. (2007). The effects of classroom mathematics teaching on students' learning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Reston, VA: National Council of Teachers of Mathematics.

Moschkovich, J. (2012). *Mathematics, the Common Core, and language: Recommendations for mathematics instruction for ELs aligned with the Common Core*. Available online http://ell.stanford.edu/

Stein, M. K., Grover, B., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455–488.

Understanding Language. (2012). *Guidelines for English Language Arts materials development*. Available online http://ell.stanford.edu/

Understanding Language. (2013). *Key principles for ELL instruction*. Available online http://ell.stanford.edu/

Walqui, A, Koelsch, N., & Schmida, M. (2012). *Persuasion across time and space: Analyzing and producing complex texts*. (Unit developed for the Understanding Language Initiative by WestEd's Teacher Professional Development Program.) Stanford University. Available at ell.stanford.edu

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The team has benefitted from the advice of several "critical friends" throughout the development of these resources. As part of the Understanding Language initiative, George Bunch provided valuable feedback on earlier drafts of the resources. Aida Walqui, Martha Castellón, Lydia Stack, and Kenji Hakuta were supportive and encouraging throughout the process. Judit Scott provided advice on vocabulary scaffolding.

APPENDIX A | POINTERS TO PROFESSIONAL DEVELOPMENT MATERIALS

A central issue in teaching ELLs mathematics is supporting classroom mathematical discussions. Below we provide pointers to Professional Development Materials (books, videos, etc.) that can be used by teachers to learn to orchestrate mathematical discussions. Although teachers can read the materials on their own, the best settings for this type of professional development would be long term study groups or professional development experiences.

Mathematical discussions

1. *Five practices for orchestrating productive mathematics discussions.* Margaret Smith and Mary Kay Stein. Corwin Press, 2011.

   This book's five practices connect students' approaches with the underlying mathematics and help teachers support productive classroom discussions.

2. *Classroom discussions: Using math talk to help students learn. Grades 1–6*. Chapin, S. C., O'Connor C., & Andersen, N. C. Math Solutions, 2003.

   This book provides a unique look into the significant role of classroom discussions in mathematics teaching in grades one through six. Five discussion strategies are introduced to help teachers strengthen students' thinking and learning and help them build connections among mathematical ideas. A valuable outline is provided to help teachers get started using talk in the classroom, plan lessons, and deal with challenges. Two case studies are also included for further insight into how teachers can use talk effectively.

3. *Classroom discussions: Seeing math discourse in action* (Multimedia Professional Learning Resource). Math Solutions.

   This set of DVDs provides pre-service and in-service instructors, coaches and facilitators with real, classroom-based video examples that illustrate the principles and practices covered in the book, *Classroom discussions: Using math talk to help students learn, grade K–6, second edition.* Ideally the three components—guide, DVD, and book—would be used together. The video examples in the DVD demonstrate how the talk tools described in the book can be used successfully in typical classrooms.

   Below are three links for examples of math classroom videos on the Math Solutions website. The first one is a first grade class in a two-way bilingual school.

http://www.mathsolutions.com/MathTalk/videos/CRD_Gr1.html

http://www.mathsolutions.com/MathTalk/videos/CRD_Gr5.html

http://www.mathsolutions.com/MathTalk/videos/CRD_Gr6.html

The full set of materials includes DVDs, a Facilitator's Guide that includes reproducible materials (for a sample see Appendix B: Talk Moves That Help Students Orient to the Thinking of Others).

Professional development resources relevant to ELLs and mathematics teaching

Coggins, D., Kravin, D., Coates, G. D., & Carroll, M. D. (2007). *English language learners in the mathematics classroom*. Thousand Oaks, CA: Corwin Press.

Teachers and math specialists will find ways to incorporate ELL supports and strategies through sample lessons that connect standards-based mathematical concepts with language development.

Celedón-Pattichis & Ramirez. N. (2012). *Beyond good teaching: Advancing mathematics education for ELLs*. Reston, VA: National Council of Teachers of Mathematics.

Through guiding principles and instructional tools, together with classroom vignettes and video clips, this book shows how to go beyond good teaching to support ELLs in learning challenging mathematics while developing language. The design of this book is interactive and requires the reader to move back and forth between the chapters and online resources at nctm.org.

Remember the Guidelines for Watching Videos of Teaching, page 9 of the facilitator's guide or Reproducible 1.1b.

REPRODUCIBLE 1.3

Professional Development Session 1.3
Talk Moves That Help Students Orient to the Thinking of Others

Discussion Questions

The videos in this session focus on talk moves that teachers use to help students orient to the thinking of their classmates: listening and trying to understand. As you view the videos in this session, you will be considering examples of a family of talk moves we call "Who can repeat?"

1. "Who can repeat?" You ask students to restate, repeat, or reformulate what another student has said.

When a student says something complex but potentially important, you may want to incorporate that into the ongoing discussion. But if students did not hear it, or were not paying attention, they will not be able to take the next step and think about it. (This family of talk moves includes examples like "Who thinks they can repeat what Steven said?" "Who would like to restate that?" or "Who could put that into their own words?") Consider this example (also on Reproducible 1.2):

Rania: The denominator size is opposite the fraction size.

Teacher: Can you give us an example? I'm not sure what you mean.

Rania: If the denominator is smaller, like four is smaller than five, the fraction will be bigger. Like one-fourth is bigger than one-fifth.

If you want to make sure that all students hear this, you can start by asking the class, "Who can repeat what Rania said?" Usually at least one student will be able to repeat at least part of what Rania said. After they do, make sure that you check back with the original student to see if that is what she intended.

Teacher: Who can repeat what Rania just said? It was a bit complicated, but it's important for us to think about it. Who can repeat? Terry?

Terry: I think she said that one-fourth is bigger than one-fifth, and four is bigger than five. No, I mean four is smaller than five.

Teacher: Is that what you meant, Rania?

Rania: Yes.

It's important to note that this should not be used as a management move: although some teachers use this move to catch students who are not listening, if you use it only in this way students may not be inclined to participate. They will be more enthusiastic if you use it in a positive way.

At the beginning of your efforts to use productive classroom talk moves, some students may resist repeating. It's important to get across that they are allowed to say, "I didn't hear," or "I didn't understand," but they must then ask the person to repeat, and then you must follow up by asking them to repeat after that.

2. Turn-and-talk: "Tell us what your *partner* said."

After a turn-and-talk, you can ask students to tell the whole class what they said to their partner. As described in Reproducible 1.2, this preparation helps reluctant students speak up. You can also use this practice to help students orient toward the thinking of others. When you ask a student to report out after a turn-and-talk, you can say, "Tell us what your *partner* said." For students who would prefer to use the airtime for themselves, this helps get across the message that all students are responsible for listening to others and for being able to repeat back what they said.

Session 1.3 Discussion Questions

- Which talk moves have you used? What have your observations been about them?
- What are the potential benefits of each move for the student who is speaking and the other students in class?
- What are the potential benefits for the teacher?
- Are there costs of each move for the students or for the teacher?
- How could the perceived challenges of these moves be approached and dealt with?

Video Clips 1.3a Discussion Questions (First Viewing)

- What did you see happening here?
- Did anything surprise you, interest you, or make an impression on you?

Video Clips 1.3a Discussion Questions (Second Viewing)

- What can you observe about the student who said the original utterance and the students who are repeating?
- How do the interactions you see provide opportunities for formative assessment?
- Do you see evidence that these interactions could support language development with younger students? with English language learners? Do you see evidence that students' learning of academic language may be served by this type of talk move?
- Do you see evidence that the classroom interactions in these video clips support more robust understanding on the part of the students?

Appendix C | Multiple Pathways for ELLs at different levels

Adapted from ELA unit

The mathematics materials were *not* specifically designed for students at different English levels but should be appropriate for students who have reached at least an intermediate level of proficiency in English (see Level 3 in the *Understanding Language ELP Framework*) and is adaptable to newcomers and advanced students.

Effective implementation of the materials assumes a teacher who both is knowledgeable about the critical role of language envisaged in the Common Core State Standards and knows how to support students' learning across the mathematics curriculum. In many cases, supporting teachers in the development of this knowledge and these set of skills will require some professional development prior to their teaching using the annotations.

Teachers can design multiple pathways for differentiating instruction so that all students can achieve at high levels. Levels of support can be designed to foster increasing levels of autonomy and independence over time. Below, as an example, options for minimal, moderate, and maximal levels of scaffolding are described for supporting students in reading word problems, using the Language of Mathematics Task Template for "Reading and Understanding a Mathematics Problem" (See Appendix D). These options engage all ELLs—and any other students in class—in mathematical reasoning and sense making with varying levels of support.

EXAMPLE

Reading and Understanding a Mathematics Problem

Materials:

-Any mathematical task that involves reading a word problem

- Language of Mathematics Task Template for "Reading and Understanding a Mathematics Problem" (See Appendix D)

Three options are presented that teachers may choose depending upon their students' level.

- Option 1: Implementation of the task with minimal scaffolding
- Option 2: Implementation of the task with moderate scaffolding
- Option 3: Implementation with maximal scaffolding.

Option 1: Implementation of the task with minimal scaffolding

Ask students to sit in heterogeneous groups of three. Distribute the task to each group.

Option 2: Implementation of the task with moderate scaffolding

Tell students that they are now sitting in Base Groups. Based on each student's English proficiency and reading level and your knowledge of the task, assign each student a number from 1 to 3. Subdivide expert groups, if needed, so that each group has no more than four students.

Option 3: Implementation with maximal scaffolding

In this option, the teacher reads each task aloud. The teacher stops at key points and asks students to talk to a partner about whether they can enter information into their chart and what information that might be. The teacher asks for student input and guides the group in their response. Collaboratively, the class works together to fill in the chart, with the teacher modeling what should be written in each cell either on poster paper or through a document camera.

Language of Mathematics Task Templates

Table of Contents

Preface

The mathematics tasks with annotations (available on the UL web site) provide examples of how teachers can use a mathematics task that is aligned with the CCSS when working with ELLs. Each annotation includes at least one Language of Mathematics task designed to support students in learning to read and understand word problems, or review vocabulary, or communicate about a mathematical problem they have solved. In this Appendix, we provide templates for five of these Language of Mathematics tasks.

Teachers can use these five templates to design and write their own Language of Mathematics tasks to fit a mathematics task of their choice.

Student materials and descriptions for the Language of Mathematics Task Templates were either adapted from the Understanding Language unit in English Language Arts (Walqui, Kolesch, & Schmida, 2012), adapted from materials used by R. Santa Cruz and H. Asturias, both professionals in teacher professional development, or recommended as "good bets" by researchers with expertise in how to scaffold vocabulary and reading comprehension. The materials draw, in large part, on papers prepared for the Understanding Language Conference at Stanford University (http://ell.stanford.edu/papers) and the ELA unit written by Walqui, Kolesch, & Schmida (http://ell.stanford.edu/teaching_resources/ela).

These templates for the Language of Mathematics tasks focus on two issues: reading word problems and using vocabulary to communicate about solutions to a mathematics problem. This is not because we think that these are the most central for learning mathematics *or* language but, instead, for several pragmatic reasons. First, teachers have often raised these two issues as the major stumbling blocks they face when teaching ELLs mathematics. Second, there were existing exemplars of activities addressing these two issues that had already been piloted with students and teachers and used in professional development work by R. Santa Cruz and H. Asturias.

In this Appendix, we provide templates for each task type with a general description for how to organize each activity. However, the best way to see how these Language of Mathematics tasks work when used with a CCSS-aligned mathematics task is to look at the following mathematics tasks with annotations on the Understanding Language website:

Elementary School | *Roger's Rabbits*

Mathematically Speaking (p.11-14)

Middle School | *Making Matchsticks*

Mathematically Speaking (p. 12-15)

High School | *Sidewalk Patterns*

Reading and Understanding a Mathematics Problem (p.14-19)

High School | *Creating Equations*

Jigsaw Reading (p. 14-29)

Reading and Understanding a Mathematics Problem (p. 21-24)

Mathematically Speaking (p. 25-29)

Tasks to Support Reading Mathematics Problems

We provide two templates for Language of Mathematics tasks to support students in learning to read and understand word problems:

1. Reading and Understanding a Mathematics Problem (*page 37*)
2. Jigsaw Reading (*page 41*)

Reading and Understanding a Mathematics Problem

Adapted from handout developed by Harold Asturias.

Purpose

The purpose of Reading and Understanding a Mathematics Problem is to support students in learning to approach a mathematics problem. It gives students tools for learning to read, understand, and extract relevant information from a problem. It also gives students practice in identifying additional information they need in order to solve the problem.

Required for use

- Handout: Student materials.
- A mathematics task using realistic quantities or a real world scenario (not necessarily actual data) that requires reading text. The task should provide some information while omitting other information necessary to solve the problem.

Structure of the activity

1. Students begin by reading or attempting to read the problem individually.
2. Students then form into pairs and work together to talk through the problem using the handout provided. There are five steps in talking through the problem together, three of which begin with reading the problem aloud.

   Step 1 Students identify what the problem is about (marbles; concert tickets; or a rectangle).

   Step 2 Students are asked to make explicit what information they need to find (a number of marbles; the numbers of two kinds of tickets, each with a different price; or the length and width of a rectangle).

   Step 3 Students answer these questions together, both orally and in writing.

   Step 4 Teacher asks a scaffolded set of questions leading to a diagram that represents both the known and unknown information about the quantities in the situation.

Step 5 Teacher asks students to try to act out the problem using real objects to represent the quantities in the situation.

3. Pairs of students should present their diagrams to the class. As they view and interpret other students' diagrams, they should add details or labels to their own.

Process outline

1. Students work individually on the problem.
2. Students form pairs and each pair shares one copy of the handout.
3. Pairs of students talk together to answer the questions in Steps 1–3 on the handout in writing.
4. Finally students try to act out the problem using physical objects to represent the quantities in the situation.

Name ____ Date _________________________

Student Materials:
Reading and Understanding a Mathematics Problem

Step 1. Read the problem out loud to a peer. Try to answer this question.

What's the problem about?

Step 2. Read the problem again. Talk to your partner about these questions:

What is the question in the problem?

What are you looking for? (Hint: Look at the end of the problem for question.)

Adapted from handout developed by Harold Asturias

Step 3. Read the problem a third time. Talk to your partner about these questions.

a) What information do you need to solve the problem? (What do you want to know?)
b) What information do you have? (What do you know?)
c) What information are you missing? (What don't you know?)
d) Draw a diagram of the problem and label all the information you know.

Information

Step 4. (If useful for this problem) Draw a diagram, act the problem out, use objects to represent the problem situation.

Adapted from handout developed by Harold Asturias

Jigsaw Reading

Adapted from the ELA unit (Walqui, Kolesch, & Schmida, 2012).
Note: Jigsaw Reading is an activity in a very early draft form and has not yet been piloted in classrooms. It may turn out to work best with longer complex mathematics text.

Purpose

This task aims to alert students to the organization of a math problem or text and the discourse and content connections that make texts flow and be predictable. For example, the structure of a math problem begins with the statement of some given information, then there may be more information, and finally there is a question or request for a solution or missing information. In this activity, students are given one piece of a problem, and they must read closely to determine where in a math problem their section fits. In the process, students begin to focus, without prompting, on how grammatical and lexical choices create cohesion and meaning within and across sentences and how larger units of text are connected to create coherence or a unity of meaning. The activity apprentices students into the type of close reading needed to understand more complex math problems and math texts.

Required for use

An ideal math problem or text for this treatment should be no longer than a half page. Initially, the sections should contain clear markers of organization that are characteristic for that type of problem. As students become more sophisticated readers of math problems they may benefit from reading and reassembling texts that are clearly organized but do not use “set” markers to signal organization.

Structure of the activity

Initially, the teacher sets out the overall purpose of the task by explaining that writers of math problems use language to connect ideas within and across paragraphs in a text. The teacher should explain that students will be given sections of a math problem that has been divided into three pieces. They will reassemble the problem by putting the pieces in order, and this will help them understand how math problems work. The teacher might introduce the task with a math problem or text that is familiar to the class.

The selected text is cut into its sections that are placed in an envelope (the number of sections determines the number of students in a group). The teacher distributes and review the directions.

Process outline

4. One student distributes the text sections randomly to the group members.
5. Each student then reads his or her section silently and tries to imagine where the segment fits into a whole: Is it a beginning? The middle? The end? What makes them think so? Students must have reasons for their thinking.
6. When everyone in a group appears to be ready, the person who thinks he or she has the first piece says, “I think I have the first piece because…” and without reading the text aloud explains what clues led to this supposition. If any other group members think they have the first piece, then they too must explain, “I think I have the first piece because…” Once the group decides what piece should go first, the person with that piece reads it aloud.
7. After hearing the piece read aloud, the group discusses whether it is indeed the first piece. If agreement is reached, the piece goes face up on the table where group members can refer to it as needed.
8. Students follow the same procedure to reconstruct the rest of the text, section by section.
9. If students feel they have made a mistake along the way, they may go back and correct it.
10. Once the whole process is finished, all group members review the jigsawed text to make sure it has been assembled correctly.
11. The teacher can facilitate a whole group discussion asking students to explain which connectors or other linguistic features helped them to ascertain the order of the sentences. The teacher can use strips of transparencies on an overhead projector or paper strips on a white board to manipulate during student explanations. Where warranted, the teacher can provide alternative ways of stating similar relationships to those from the math problem.

Tasks to Support Vocabulary for Mathematical Communication

We provide three templates for Language of Mathematics tasks to support students in learning to use vocabulary to communicate about their solutions to a mathematics problem.

1. Vocabulary Review Jigsaw (*page 44*)
2. Mathematically Speaking (*page 47*)
3. Vocabulary Pieces, Roots, and Families (*page 50*)

Overall, there are several ways to work with vocabulary, including introducing, using and reviewing terms. It is important to note that vocabulary need not be pre-taught or introduced in isolation but instead it should be included in activities that involve cognitively-demanding work—for example, reasoning, sense making, explaining and comparing solutions to a mathematics problem. Thus, decisions to "pre-teach" vocabulary should be carefully considered and should include activities where students *actively* solve and discuss their solutions to a mathematics problem.

When introducing new vocabulary, it is useful to first give students the opportunity for an engaging and successful problem-solving experience. Having grounded meanings by first solving a mathematics problem and discussing their solutions, the teacher can then label, define, and review technical terms.

Importantly, then, the purpose of vocabulary work should not be *primarily* for students to learn English vocabulary but to provide access to the mathematical work. The purpose of vocabulary work should be to provide opportunities for students to use mathematical language to communicate about how they solved a problem, describe their reasoning, explain why a solution or step works, and/or justify a claim, etc. Students will then learn English vocabulary as they engage in solving and discussing substantive mathematics problems. Lastly, tasks and activities that engage students in learning vocabulary should address several types of terms.

In this example: *Jane, Maria, and Ben each has a collection of marbles. Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles. Find how many marbles Maria has.*

- Contextual or colloquial terms and phrases, for example, *marble*.
- Vocabulary specific to mathematics, for example, *more marbles than* or *2 times as many marbles as*.
- General academic language, for example, *find*, *describe*, *analyze*.

Vocabulary Review Jigsaw

Adapted from the ELA unit (Walqui, Kolesch, & Schmida, 2012).
Note: Vocabulary Review Jigsaw is an activity in a very early draft form. The activity has not yet been piloted in mathematics classrooms.

Purpose

This task gives students an opportunity to review vocabulary or terms. Students work in groups of four to combine the clues held by each member and try to guess the 12 target words. It is important to recognize that this task is intended to be used not to teach vocabulary but to *review* vocabulary.

Required for use

The teacher selects 12 key vocabulary items or terms that the students have been introduced to within a task, lesson, or unit. The teacher prepares five cards—four to be used in the jigsaw, and one for the answer key.

There are two ways to prepare the jigsaw cards (Version I and Version II). The teacher provides four clues that will help student identify each vocabulary word. In Version I, the clues for each word fall into four categories. Three of the categories are very simple: (a) the first letter, (b) the number of syllables, and (c) the last letter. The fourth clue, (d), is a working definition of the term. The definition is not one from the dictionary; rather, the definition should be written by the teacher, using knowledge stressed in class.

In Version II, all the clues are meaningful. Clue A should be the broadest, opening up many possibilities. Clue B, while narrowing the selection of an answer, should still leave it quite open. Clue C should narrow the possibilities further. Clue D should limit the possibilities to the target word.

Structure of the activity

Students need to be in groups of four. The teacher explains to students that they will participate in a fun way to review vocabulary. Then the teacher models the activity. It should be stressed to students that the activity is collaborative and that all four clues (A, B, C, and D) must be heard before the group can guess the vocabulary word. The teacher should prepare a small jigsaw to use as an example, modeling the process with a key term the students have learned in a previous unit. For example, the teacher might choose the term "hyperbola" and prepare four index cards with the following clues:

A: The first letter is "h."
B: There are four syllables.
C: The last letter is "a."
D: The word means _____________ .

The sample cards show two examples for "axis" and "intercept."

Four students should work together to model for the class, with each student reading only his or her assigned clue.

CARD A

1. The word starts with the letter A.
2. The word starts with the letter I.

CARD C

1. The last letter in this word is x.
2. The last letter in this word is t.

CARD B

1. This word has TWO syllables.
2. This word has THREE syllables.

CARD D

1. It means the vertical and horizontal lines that determine the quadrants of a coordinate plane.
2. It means the point at which a line, curve, or surface intersects an axis.

Student's Answer Sheet (sample)

1. ______________________________
2. ______________________________

Process outline

1. Students sit in groups of four.
2. Students number a piece of paper 1 to 12, down the left hand side.
3. The student with card A selects the number he or she would like to read, and all group members then circle the number on their answer sheets.
4. Each student reads their clue for that number, in order: A, B, C, and D.
5. After all four clues have been read, the students try to guess the word or term.
6. Students write their answer in the appropriate line on their answer sheet.
7. After three turns, students pass their cards to the person on their right so that all four students have a chance to read all four clue cards.
8. When a group has completed the jigsaw, one member asks for the answer key, and the group checks its answers, taking note of any terms that require additional study.

Mathematically Speaking

Adapted from the work of R. Santa Cruz for the Understanding Language Initiative.

Purpose

This task gives students the opportunity to solve a problem and then explain and discuss how they arrived at their solution using targeted vocabulary. The activity is used for vocabulary review or guided practice.

The purpose of this task is to alert students to important vocabulary and terms *during* contextualized mathematics activity. Students are asked to listen for, track, and describe vocabulary they used while their group is solving a mathematics problem. It is crucial that students do this vocabulary work *after* they solve a mathematics problem that grounds the meanings for words.

Students will use everyday words while solving a mathematics problem or in early rounds of talking about their solutions with other students or the teacher, and they should not be corrected. Instead the teacher can provide more formal mathematical terms later during a whole class discussion.

Note that developing academic language involves more than just learning the target or specialized vocabulary of a unit or chapter. Comparative structures such as “twice as many, 3 less than 7” are syntactic structures that students also need as they use the target vocabulary of mathematics tasks.

Required for use

The teacher selects a set of key terms that the students have been introduced to within a task, lesson, or unit. The teacher prepares a chart or organizer for students to use.

Structure of the activity

All students independently complete both mathematical tasks or problems. They may use more space on a separate piece of paper, but must show their work

Students form pairs and and each pair should get one copy of the Mathematically Speaking chart.

Target vocabulary words are written on the chart in the left column. For lower grades, the teacher can fill in the words. The two students write their names across the top. One student explains how they solved the mathematics problem to the other student as the other student uses a checkmark on the chart to record each time a target word is used in the explanation. The other student then takes a turn doing the same.

Students can keep talking until all target words have been used.

This activity may be used for practice or assessment after students have worked on a mathematics problem and teachers have provided instruction or modeled the formal usage of the target vocabulary.

To support students in refining their descriptions and explanations, students can ask each other these questions:

- Did my explanation make sense?
- Do you have any questions about what I did?
- Do you have any questions about why I did this?

To focus on their mathematical reasoning, students can ask each other these questions:

- What did you do to solve the problem or find an answer?
- Why did you do that step?
- Why is that step is justified mathematically?" or "What is a mathematical reason for that step?"

Process outline

1. Student pairs are formed.
2. Target vocabulary words are written on the Mathematically Speaking chart in the left column.
3. For lower grades, the teacher or volunteer may fill in the words.
4. The two students write their names across the top.
5. One student explains their solution to the other student as he or she writes a check on the chart each time a target word is used in the explanation.
6. Students keep talking until all target words have been used.
7. Students keep talking until all target words have been used.

Student Materials:
Mathematically Speaking

Date ___________________

Partner Names ___________________________ & ______________________________

Task Name ____

1. Solve the problem. Show your work
2. Explain your thinking, strategies, and solution to your partner. Use the target words in your explanation.
3. Listen to your partner's explanation and make a tally for each time he or she used the target vocabulary.

Explain how you solved the problem.	Explain how you solved the problem.
Problem 1	Problem 2

Vocabulary Words	Tally: How many times used
Example: variable	III

Vocabulary Words	Tally how many times used
Example: Constant	IIII

Vocabulary Pieces, Roots, and Families

Adapted from activities suggested by Judith Scott.
Note: This activity has not yet been piloted in mathematics classrooms.

Task One | Looking at word pieces in "equation"

Introduce the concept of a morpheme: Words made up of smaller pieces of meaning.

Ask students:

- Do they know what two parts make up the word *equation*? *equate* + *ion*.
- Do they know what the suffix -*ion* does? (This suffix turns the verb form of a word into a noun form.)
- The teacher asks students to "mine their brains" to come up with some other examples of words where *ion* turns a verb into a noun.
- The teacher records this on chart paper.
- The teacher asks students to collect examples of -*ion* words throughout the next 24 hours and to add examples to the chart.

  For example: define, definition;

  organize, organization;

  explore, exploration;

  equate, equation.

  This is a highly productive pattern to learn in English.

Task Two | Looking at the root word *equate*.

- The teacher asks students what words they know that might be related in meaning to the root word for equation. Students can use WordSift.com or visualthesaurus.com to come up with ideas.
- The teacher asks students to collect words that they identify as related. For example: *equal*, *equivalent*, *equalize*.
- The teacher points out that all these words come from the Latin *aequare* which means to "make even or uniform, make equal" (p. 18).

Note that usage differs. Sometimes equations are considered to be expressions and sometimes they are not. In the Common Core State Standards, equations are not considered to be expressions.

Note also that the two expressions in an equation may not always be equal to each other. For example, an equation may be true for some values of its variables and false for others. Or an equation may simply be false, e.g., 0 = 1.

- The teacher asks students, working in groups, or pairs, to pick two words from the list of identified words. Writing on sentence strips, students should explain how the meaning of each word relates to the Latin root.
- The teacher provides some example of good explanations:

  Equation: An equation is a statement that two expressions are equal to each other.

  Equivalent: If two algebraic expressions are equivalent, it means that they always have the same value.

  Equivalent: If two numerical expressions are equivalent, it means that they have the same value. That is, they are equal to the same number (or both undefined).

  Students may use everyday language to explain meanings of words in a way that makes sense to them. The teacher should have different groups read out the explanations until all the words are used.

Task Three | Finding Words in a Family

- Ask students to try to collect other words in the "equal" family.
- Post a chart that can be added to, or have students keep a record of these words in a "word catcher" notebook.

References

Nagy, W. E., & Scott, J. A. (2000). Vocabulary processes. In R. Barr, M. L. Kamil, P. B. Mosenthal, & P. D. Pearson (Eds.), *Handbook of reading research* (Vol. 3, pp. 269–284). New York: Longman.

Scott, J. A., & Nagy, W. E. (2004). Developing word consciousness. In J. F. Baumann & E. J. Kame'enui (Eds.), *Vocabulary instruction: Research to practice* (pp. 201–217). New York: Guilford.

Walqui, A. Koelsch, N. & Schmida, M. (2012). *Persuasion across time and space: Analyzing and producing complex texts*. (Unit developed for the Understanding Language Initiative by WestEd's Teacher Professional Development Program.) Stanford University. Available at ell.stanford.edu

Roger's Rabbits

Level: Elementary School

Version 10.28.13

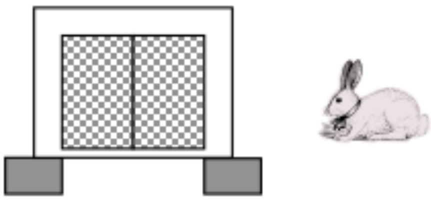
This task gives students the chance to:

- work with patterns
- work with tables

Table of Contents

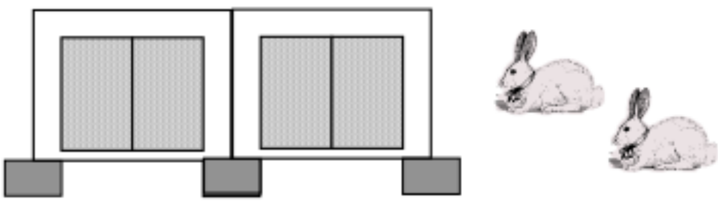
Name ____________________________________

Roger keeps pet rabbits. He keeps them in a row of rabbit hutches. The hutches are on blocks so that they don't get damp.



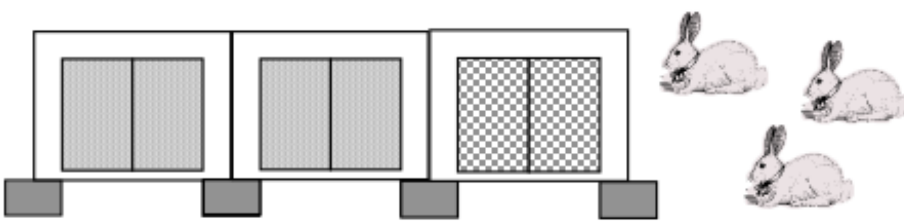
This is hutch #1.

It is for one rabbit.
It has 2 doors and 2 blocks.



This is hutch #2.

It is for two rabbits.
It has 4 doors and 3 blocks.



This is hutch #3.

It is for three rabbits.
It has 6 doors and 4 blocks.

1. Fill in the empty spaces in the table below.

Hutch	1	2	3
Number of doors			
Number of blocks			

2. Roger says: "To get the number of doors for a hutch, I take the hutch number and double it."

a. Does Roger's rule work for hutches #1, #2, and #3?

Use words, tables, diagrams, expressions, or equations to explain your answer.

b. According to Roger's rule, how many doors does hutch #12 have? ________________

Explain how you figured this out.

____

____

3. Sara says: "To find the number of blocks for a hutch, I take the hutch number and add 1."

Does Sara's rule work for hutches #1, #2, and #3?

Use words, tables, diagrams, expressions, or equations to explain your answer.

Roger's Rabbits | Annotations

Core Idea

Roger's Rabbits asks students to identify whether finite sequences of numbers follow given rules and to extend those sequences according to those rules. Note that a finite sequence such as 2, 4, 6 does not determine exactly one pattern. For example, it could be seen as part of a "multiples of 2" pattern and continued as 2, 4, 6, 8, . . . Or, it could be seen as part of a repeating pattern such as 2, 4, 6, 2, 4, 6, 2, 4, 6, . . .

Note that the two different kinds of shading on the doors do not have mathematical meaning.

Common Core State Standards for Mathematical Content
http://www.corestandards.org/Math/Content/4/OA

Grade 4, Operations and Algebraic Thinking
(p. 29)

Use the four operations with whole numbers to solve problems.

3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

With regard to this standard, the draft Operations and Algebraic Thinking Progression for the CCSS comments, "notice that the Standards do not require students to infer or guess the underlying rule for a pattern, but rather ask them to generate a pattern from a given rule and identify features of the given pattern."

Common Core State Standards for Mathematical Practice
http://www.corestandards.org/Math/Practice

SMP.1. Make sense of problems and persevere in solving them.	. . . Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships. . . . They can . . . identify correspondences between different approaches. . . .
SMP.3. Construct viable arguments and critique the reasoning of others.	Mathematically proficient students . . . justify their conclusions, communicate them to others, and respond to the arguments of others. . . .
SMP.6. Attend to precision.	Mathematically proficient students try to communicate precisely to others. . . . They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. . . .
SMP.7. Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. . . .
SMP.8. Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts.

Common Core State Standards for ELA/Literacy (grade 4)
http://www.corestandards.org/ELA-Literacy/SL/4

Key Ideas and Details (p. 14) http://www.corestandards.org/ELA-Literacy/RI/4/1	1. Refer to details and examples in a text when explaining what the text says explicitly and when drawing inferences from the text.	. . . Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships. . . . They can . . . identify correspondences between different approaches. . . .

Craft and Structure (p. 14) http://www.corestandards.org/ELA-Literacy/RI/4/4	4. Determine the meaning of general academic and domain-specific words or phrases in a text relevant to a grade 4 topic or subject area.	Mathematically proficient students . . . justify their conclusions, communicate them to others, and respond to the arguments of others. . . .
Integration of Knowledge and Ideas (p. 14) http://www.corestandards.org/ELA-Literacy/RI/4/7	7. Interpret information presented visually, orally, or quantitatively (e.g., in charts, graphs, diagrams, time lines, animations, or interactive elements on Web pages) and explain how the information contributes to an understanding of the text in which it appears.	Mathematically proficient students try to communicate precisely to others. . . . They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. . . .
Speaking and Listening (p. 24) http://www.corestandards.org/ELA-Literacy/SL/4/2	2. Paraphrase portions of a text read aloud or information presented in diverse media and formats, including visually, quantitatively, and orally.	Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. . . .

Suggestions

Before students attempt Question 1, teachers might:

1. Read the problem as a class.
2. Think, Talk, Share: Have the students talk in pairs to discuss the meaning of the task:
    - Have students look at the pictures and describe a hutch.
    - Negotiate for meaning as students come to common understandings or agreements regarding the use of formal and informal mathematical language. For example, students may need to agree on the following:

      What constitutes a hutch? Is it a singular space or multiple spaces? Although an illustration is provided, students may consider each pair of doors as one hutch, when actually each row is considered one hutch.
3. Ask students to write down three things they agree on, using words, phrases, pictures and/or diagrams on poster-sized paper. Put these up in front of the room. Have a reporter from each group to read their agreements.
4. Show the pictures without the words and ask students to describe what they see to each other.

   Ask students:

    - "What did we notice about hutch #1?"
    - "Hutch number 1 has two blocks, hutch #2 has three blocks. Why does it *not* have four blocks?"
    - "Tell me something you notice about hutch #3." Have students begin with "I/We notice . . . "; "We think . . . "; or "I see . . . "
    - Write responses on chart paper and post.

1. Fill in the empty spaces in the table below.

Hutch	1	2	3
Number of doors			
Number of blocks			

To assist students in completing the table:

1. Have each student complete the table independently, then share the results with a peer.*
2. Ask students to explain how they came to their particular results and to adjust their responses based on the new information, if necessary. This allows for more talk, justification, and peer teaching and learning. This also encourages peer-to-peer teaching and learning as well as peer monitoring.
3. Ask students to describe any patterns they see in the table to another student. This is an opportunity for them to use words, the table (possibly augmented by more rows or columns, or annotations), diagrams, expressions, or equations in describing any regularities that they see. In particular, this is an opportunity to see repeated calculations, e.g., repeatedly adding 1 to get the number of blocks for the next hutch or repeatedly adding 2 to get the number of doors for the next hutch.

*In many cases ELLs are reassured by the illustrations that support their response and depend on them when they are uncertain about the language. Often they receive lower scores for the language expression rather than the mathematical understanding. In these cases, teachers might give two scores —one for writing and another for mathematics.

2. Roger says: "To get the number of doors for a hutch, I take the hutch number and double it."

a. Does Roger's rule work for hutches #1, #2, and #3?
Use words, tables, diagrams, expressions, or equations to explain your answer.

b. According to Roger's rule, how many doors does hutch #12 have?

Explain how you figured this out.

Students are being asked to compare the result of applying

Roger's rule for 1, 2, and 3 with the entries that they made in their tables. Second, they are asked to apply the rule to a hutch whose picture is not shown. To help clarify the rule consider the following:

1. Ask a volunteer to re-read Roger's rule aloud to the class.
2. Have students work in pairs and share ideas about how they can show how they answered the question.
3. Allow students to build or illustrate the hutches/doors/rabbits as they figure out how many of each will be needed for hutch #12. This is especially helpful for ELLs.*

To assist students having difficulty with this task, use patterned language that supports the correct results.
For example:

According to Sara's rule, hutch number four has five blocks,

According to Sara's rule, hutch number five has six blocks,

According to Sara's rule, hutch number six has ________________ ? The whole class should respond, "Seven blocks."

Or, for a second example:

"I agree with _________________'s response because ______________________________."

"I disagree with _______________'s response because ______________________________."

Here there is less scaffolding, which allows students with more advanced fluencies and mathematics understanding to do the "heavy lifting" in the task.

*Note that according to the CCSS, "Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)"

Language of Mathematics Task | Mathematically Speaking

Adapted from R. Santa Cruz (2012) for the Understanding Language Project

This Language of Mathematics task is provided as a resource to be used, revised, and combined to fit a variety of lesson plans. The overall goals are to minimize direct instruction and introduction by the teacher, and instead provide structure so that the students can grapple with the questions themselves. Students first work alone, then in pairs or small groups, and finally in a whole class discussion while always focusing on their mathematical reasoning. This cycle provides ELLs with the opportunity and time to think, practice speaking in pairs or small groups, and thus be better prepared to participate in a whole class discussion or a presentation of their reasoning. Students should be encouraged to describe not only *what* they are doing but also, more importantly, *why* they are doing it. Teacher questions and whole class discussions should focus on describing, refining, and comparing students' mathematical reasoning.

As students work on Roger's Rabbits, they will use a variety of words to talk about their reasoning.

To encourage the use of relevant mathematical vocabulary, phrases, or statements, model using these throughout the lesson.

A partial list of vocabulary words and phrases that may be used in this task is below.

General	Mathematical	Statements
Table	Pattern	Twice as many . . .
Hutch	Extend	One more than . . .
Explain	Number	One less than . . .
Increase	Double	
Blocks	Twice	
	Rule or Rules	
	Multiply	

Note that words like *table*, *blocks*, and *rule/rules* have multiple meanings. Check to see if students understand their meanings in this context. It is not necessary to use all the words, but attempt to use as many of the mathematical terms as possible.

Structure of the activity

1. After students have completed Roger's Rabbits, assign partners and have them discuss how they solved the problems by doing the Mathematically Speaking task.

2. After the discussions, ask the students to create a class poster of words and phrases that they listed on their Mathematically Speaking charts. The class poster need not look like the example shown here and may include words, phrases, diagrams, expressions, and equations.

3. Note that it is not necessary to use all the words in one session. If the students leave out critical words such as *extend* or *increase*, add the words to the list as the lesson progresses.

Keep this poster on display to provide students with visual cues, connections to similar tasks, and reminders to use the vocabulary.

Words & Phrases Used by Student Groups

- more than
- pattern
- number
- explain
- double
- twice

This type of poster might be useful if it listed synonyms, e.g., "twice," "two times as many," and associated terms, e.g., "double."

Mathematically Speaking

Date ____________________

Partner Names ______________________________ & ______________________________

1. Solve the problem individually. Show your work. If needed, use the back of the page.
2. Explain your thinking, strategies, and solution to your partner. Use the target words in your explanation.
3. Listen to your partner's explanation, and make a tally for each time he or she uses the target vocabulary.

Explain how you solved the problem.	Explain how you solved the problem.
Problem 1	Problem 2

Vocabulary Words	Tally: How many times used
Example: rabbit(s)	IIII II
blocks	
increase	
next	
pattern	
rule(s)	
table	
think	
+ twice as many	

Vocabulary Words	Tally: How many times used
Example: each	III
double	
extend	
multiply	
number	
plus	
+ one more than	

If you found other helpful words, list them here:

Mathematically Speaking

Date ____________________

Partner Names ____________________________ & ______________________________

1. Solve the problem individually. Show your work. If needed, use the back of the page.
2. Explain your thinking, strategies, and solution to your partner. Use the target words in your explanation.
3. Listen to your partner's explanation, and make a tally for each time he or she uses the target vocabulary.

Explain how you solved the problem.	Explain how you solved the problem.
Problem 1	Problem 2

Vocabulary Words	Tally: How many times used
Example: rabbit(s)	IIII II
+ twice as many	

Vocabulary Words	Tally: How many times used
Example: each	III
+ one more than	

If you found other helpful words, list them here:

____

Understanding Language

Language, Literacy, and Learning in the Content Areas

Understanding Language aims to enrich academic content and language development for English Language Learners (ELLs) by making explicit the language and literacy required to meet the Common Core State Standards (CCSS) and Next Generation Science Standards (NGSS).

http://ell.stanford.edu

Making Matchsticks

Level: Middle School

Version: 7.2.13

This task is intended to help teachers assess how well students are able to:

- Interpret a situation and represent the variables mathematically.
- Select appropriate mathematical methods.
- Interpret and evaluate the data generated.
- Communicate their reasoning clearly.

It is accompanied by a formative assessment lesson.

Table of Contents

Name ________________________________

Making Matchsticks

Matchsticks are rectangular prisms of wood measuring approximately:

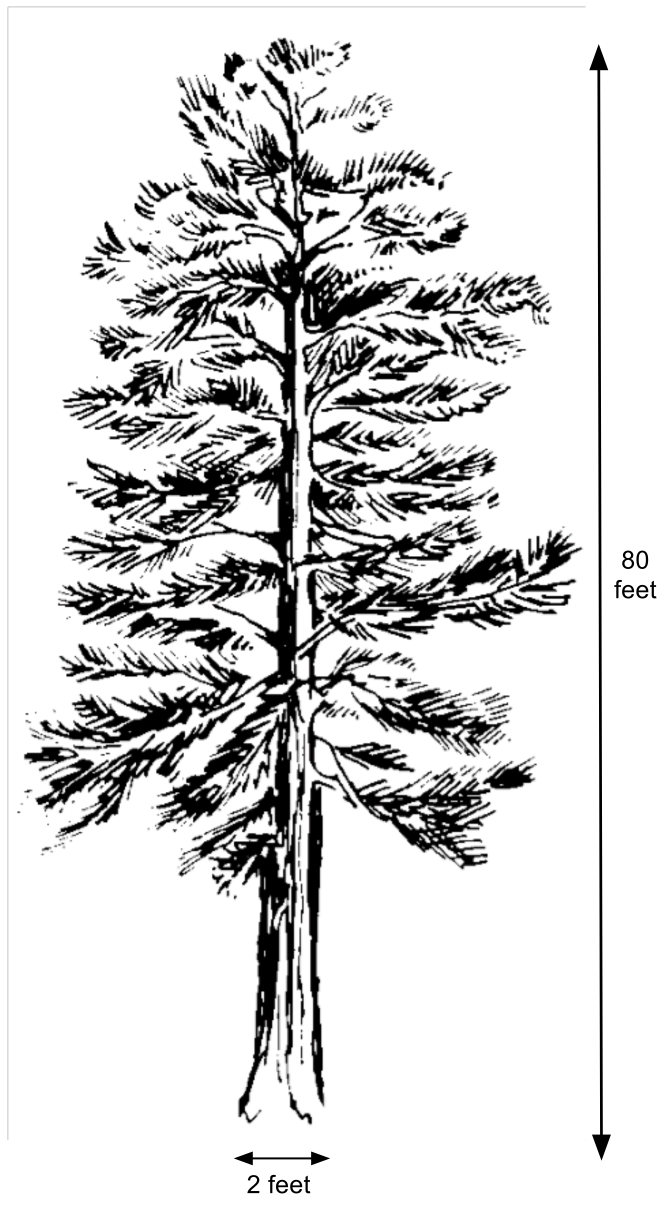
$\frac{1}{10}$ inch by $\frac{1}{10}$ inch by 2 inches

Matchsticks are often made from pine trees.

Estimate how many matchsticks can be made from this tree.

You may find some of the information given on the formula sheet helpful.

Explain your work carefully, giving reasons for any choices you make.



Making Matchsticks | Annotations

Core Idea

This task and the associated formative assessment lesson afford opportunities for students to develop strategies for solving problems. Students select formulas from a sheet of geometric formulas in order to model a situation. They interpret given data, make approximations, and communicate their reasoning in verbal and written form. Students analyze and critique solutions developed by others.

Note that although the task was designed to have students select volume formulas from a list, students are expected to know these formulas by the end of grade 8. This task draws on understandings of rate and proportional reasoning (a focus of grades 6 and 7), geometric measurement and volume which begins with right rectangular prisms in grade 5, and is extended in grades 6, 7, and 8. It is an opportunity to build toward the high school number and quantity standard of interpreting units consistently in formulas.

Common Core State Standards for Mathematics
http://www.corestandards.org/Math/Content/8/G

Grade 8, Geometry (p. 56)

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Common Core State Standards for Mathematical Practice
http://www.corestandards.org/Math/Practice

SMP.1. Make sense of problems and persevere in solving them.

. . . Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships. . . . They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. . . .

SMP.3. Construct viable arguments and critique the reasoning of others.	Mathematically proficient students . . . justify their conclusions, communicate them to others, and respond to the arguments of others. . . .
SMP.4. Model with mathematics.	Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. . . . They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
SMP.6. Attend to precision.	Mathematically proficient students try to communicate precisely to others. . . . They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. . . .
	Common Core State Standards for ELA/Literacy http://www.corestandards.org/ELA-Literacy
Grade 6, Writing, Text Types and Purposes (p. 42)	1. Write arguments to support claims with clear reasons and relevant evidence.
Grade 6, Speaking & Listening, Comprehension and Collaboration (p. 49)	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grade 6 topics, texts, and issues, building on others' ideas and expressing their own clearly.
Grade 6, Speaking & Listening, Comprehension and Collaboration (p. 49)	2. Interpret information presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how it contributes to a topic, text, or issue under study.
Grade 6, Speaking & Listening, Presentation of Knowledge and Ideas (p. 49)	5. Include multimedia components (e.g., graphics, images, music, sound) and visual displays in presentations to clarify information.

Grade 6, Reading Standards for Informational Text, Integration of Knowledge and Ideas (p. 39)	7. Integrate information presented in different media or formats (e.g., visually, quantitatively) as well as in words to develop a coherent understanding of a topic or issue.

Comments

Purpose of the task. Making Matchsticks is part of a MARS Formative Assessment Lesson (FAL), which can be downloaded here: http://map.mathshell.org/materials/lessons.php?gradeid=23.

In these FALs, initial individual student work on the task should not be supported or scaffolded by the teacher because their work is intended to produce evidence of their current knowledge. This assessment of students' prior knowledge will be used as the basis for the lesson.

Using the task with ELLs. Although students' mathematical work should not be supported or scaffolded, teachers will gain significantly more information about students' prior knowledge of the mathematics needed for the task if students comprehend what they are being asked to do in the task. In other words, although an unsupported attempt at the task by an ELL student will generate evidence of how well the student understood the task instructions, the attempt is less likely to inform a teacher about the mathematics the student would bring to the task with a clear understanding of what was being asked. Because of this, even if the task is to be used for individual assessment, we recommend that the teacher provide some support for ELLs for the individual work on the initial task for MARS Formative Assessment Lessons. The sole purpose of this support should be to give students access to the task—not directly to the mathematics they need to do the task, but rather to a clear understanding of what they are being asked to do.

Outline of the lesson. The MARS student materials include the task, a formula sheet, and a questionnaire called How Did You Work? After students have worked on the task individually (in class or as homework), they discuss the task and various responses (as described in detail in the MARS materials). Each small group of students shares a large sheet of paper for making a poster and copies of sample responses.

Here is the outline as given in the MARS materials:

1. Before the lesson, students tackle the problem individually. You assess their responses and formulate questions that will prompt them to review their work.

2. At the start of the lesson, students respond individually to the questions set, and then work in groups to combine their thinking and produce a collaborative solution in the form of a poster.

3. In the same small groups, students evaluate and comment on some sample responses. They identify the strengths and mistakes in these responses and compare them with their own work.

4. In a whole-class discussion, students explain and compare the solution strategies they have seen and used.

5. Finally, students reflect on their work and their learning.

The suggestions and the Mathematically Speaking task in these annotations are intended to supplement the lesson described in this outline. The suggestions supplement parts 1, 2, 3, and 5. The Mathematically Speaking task supplements the poster presentations in part 4.

Some potential challenges in the task are that students might:

- not take into consideration the different units used for toothpick and tree measurements.
- not understand and apply the various formulas for volume.
- misplace the decimal.
- envision the tree as two-dimensional.

For more potential challenges, see the table of possible responses in the next section.

Suggestions

The minimal support we recommend for using Making Matchsticks as an individual assessment is to ask students to mark the task, replacing “inch” by “in” and “80 feet” by “80 ft,” and to identify the objects on the upper right as matchsticks. This support is intended only to present the question and the given information more clearly, and not to add information or suggest a solution path.

Before students work on Making Matchsticks. Instructions from the teacher should be clear, direct, and concise. The direction in this activity might be reduced to the following:

- Estimate how many matches can be made from the wood in this tree.
- Use the relevant information on the formula sheet. It will help you find some answers.

- Read the task, and show all your work. Showing your work helps me understand your reasoning (thinking).
- It is important that your work is organized and presented in a clear manner (way).

Before the lesson: Assessing students' responses on Making Matchsticks. The Common Issues table below is copied and pasted from the online teacher materials for the MARS lesson Making Matchsticks; additional suggestions and prompts from the Understanding Language Project are given below this table.

Common issues:	Suggested questions and prompts:
Student has difficulty getting started	• What do you know? What do you need to find out? • How could you simplify the problem?
Student ignores the units For example: The student calculates the volume of a matchstick in cubic inches and the volume of the tree trunk in cubic feet.	• What measurements are given? • Does your answer seem reasonable if you consider the size of a matchstick compared to the size of a pine tree?
Students makes incorrect assumptions For example: The student multiplies the volume of the tree trunk in cubic feet by 12 and assumes this gives the volume of the tree trunk in cubic inches.	• Can you explain why you have multiplied by 12? • When you figure out a volume how many dimensions do you multiply together? How does this calculation effect how you convert the volume from cubic feet to cubic inches? • Can you describe the dimensions of the tree in inches? What do you notice?
Student uses an inappropriate formula For example: The student calculates the *surface area* of a rectangular prism from the dimensions given for the tree.	• Does your choice of formula make good use of all the wood in the tree trunk? • Is this the best model for a tree trunk? • What is the difference between area and volume?
Students' work is unsystematic	• Would someone in your class who has not used this method be able to follow your work? • Can you describe your method as a series of logical steps?
Students' work is poorly presented For example: The student underlines numbers and it is left to the reader to work out why this is the answer as opposed to any other calculation.	• Can you explain each part of your solution? • What does each of these calculations represent? • Can you justify the choices you have made?
Student has difficulties when substituting into a formula For example: The student multiplies the radius by 2, rather than squaring, when using the formula for the volume of a cone/cylinder. Or: The student substitutes diameter rather than radius into the formula for the volume of a cone/cylinder.	• What is the difference in meaning between $2r$ and r^2? • Does your answer seem reasonable? • How can you check your work against the information given in the problem?

Students' work is incomplete For example: The student does not divide the volume of the tree trunk by the volume of a matchstick.	• What do your calculations represent? • Have you found out how many matchsticks can be made from the tree?
Student rounds to one or more decimal places	• Why won't part of a matchstick count in your estimate?
Student completes the task	• How can you check that the method you have used has given a reasonable estimate? • Can you try a different method to check your answer? • What assumptions have you made?

Below are additional issues not found in the existing Common Issues table for this lesson, together with suggested questions and prompts. Note that these provide considerably more scaffolding than the prompts above.

Student attempts to use proportional reasoning to develop a solution path but gets confused.	• What is the rate of inches per foot? • What is the rate of cubic inches per cubic foot? • How can you use these rates to explain your solution to the problem?
Student struggles to make sense of linear, square, and cubic units.	• Draw three figures: first, draw a length of 1 inch; second draw a square with 1 inch side lengths; third, draw a cube with 1 inch side lengths. Then carefully label the side lengths in each drawing. • (If a student does not understand the exercise above, model each drawing with 1 inch, then ask the student to do the exercise with 1 foot. Then ask how many linear inches are in each linear foot, how many square inches are in each square foot, and how many cubic inches are in each cubic foot.)
Student has trouble getting started. **(Note:** this is the entry in the first row of the existing table; to the right is an additional prompt for this entry.)	• Re-read the problem aloud to your partner(s).
Student ignores the units. **(Note:** This is the entry in the second row of the existing table; to the right are an additional prompt and question that belong between the two existing questions.)	• What is being measured in this problem? • Try to explain to your partner(s) how the measurements you are given can help you find the measurements you need in order to solve the problem.

Beginning of the lesson: Reviewing individual solutions. Include a printed list of questions on which students can reply.

Some possible sentence starters for review process:

- At first I thought . . .
- I realized that . . .
- However . . .

- Now, I see that . . .
- Next time, I will . . .

Whole class discussion after students have discussed sample approaches. Possible questions to ask:

- Which approach or paper made the most sense to you? (Like and dislike may not be the best language to use here.) In which way(s)?
- Which approach was difficult to understand? Why?

At the end of the lesson or for homework. Possible questions to ask:

- Think about the different methods we used in this lesson.
- What was the most important thing you learned from your work today?
- Write about one thing that you are wondering about, find unclear, or need help with.
- Share your writing with another student.

Language of Mathematics Task Mathematically Speaking

Adapted from R. Santa Cruz (2012) for the Understanding Language Project

This task is to be used with the Formative Assessment Lesson associated with Making Matchsticks during group creation and presentations of posters.

The Language of Mathematics task is provided as a resource to be used, revised, and combined to fit a variety of lesson plans. The overall goals are to minimize direct instruction and introduction by the teacher, and instead provide structure so that the students can grapple with the questions themselves. Students first work alone, then in pairs or small groups, and finally in a whole class discussion while always focusing on their mathematical reasoning. This cycle provides ELLs with the opportunity and time to think, practice speaking in pairs or small groups, and thus be better prepared to participate in a whole class discussion or a presentation of their reasoning. Students should be encouraged to describe not only *what* they are doing but also, more importantly, *why* they are doing it. Teacher questions and whole class discussions should focus on describing, refining, and comparing students' mathematical reasoning.

Purpose

This task gives students practice tracking and interpreting target vocabulary words used by their peers during prepared group presentations. Also, it supports students in producing language, since students will be encouraged to prepare and give presentations that incorporate correct use of target words. It is not intended to give students access to the mathematics of the task during the central work on the problem, but rather to provide opportunities for using and understanding key terms when students summarize their work during poster presentations after the central work is complete.

It is crucial that students do this vocabulary work *after* they solve the problem that grounds the meanings for words. Students are likely to use everyday words in their talk while solving a problem in groups and should not be corrected. Instead, the teacher can provide guidance on more formal mathematical terms during a whole-class discussion by providing instruction on the mathematics while modeling the use of more formal vocabulary and ways of talking.

Developing academic language is more than just learning the target or specialized vocabulary of a unit or chapter. Comparative structures such as “one third of,” “at least,’” or “twelve times the amount” are the kinds of syntactic structures that students need to understand and use as they describe their work or make a presentation about their solution for a mathematics task.

Required for use

- Materials for MARS lesson Making Matchsticks.
- Mathematically Speaking tally charts with target vocabulary words.

Process details

1. Before using the Mathematically Speaking tally chart, students spend a couple of minutes reviewing their individual solutions to Making Matchsticks (completed in advance of the lesson). Use the following prompts to help students prepare to work with their peers:

    - Summarize the steps that you took to solve the problem.
    - What part of your solution are you most sure about? What part are you least sure about?

    Students can share their responses with their peers and reach a consensus as to the “best” solution to the problem. Together, students create posters displaying this joint solution to the problem.

2. After the pairs or groups create the posters, their members present to each other or to the same group (e.g., one member of a pair or group presents to the other members) using the tally sheet to help each other insert the academic language appropriately. The use of recording devices (e.g. iPods, iPads, tape recorders) can help facilitate this process as students can initially record what they want to say and replay their practice presentations. This provides a “safer” way for students to learn how to use academic language and helps ensure student success when they present in front of the whole class.

1. When pairs or groups are finished with their posters, they describe their posters to the whole class. Each student gets one copy of the chart with target vocabulary words entered. As each group describes their poster, each listening student tallies on the chart each time a target word is used. If a target word is not used, the listening students should encourage the presenters to keep

talking, by asking questions or requesting clarifications, until all target words on the list have been used.

2. Students may add words to their chart that come up in their explanations, and then share these with the class at the end of the activity.

3. The lesson continues as described in the MARS lesson outline, with the collaborative analysis of sample student responses.

Process outline

- Each student receives a copy of the handout with the tally chart.
- Each group presents their poster, while other students mark each use of the target vocabulary words on the tally chart, and add words as needed.
- Students are asked to read the list of targeted words silently before the presenters start.
- Stress how important it is for the listeners to pay attention to presenters and listen for targeted words.
- Groups begin their presentations.
- At the end of all presentations, students go over any words they may have added to the given list.

Mathematically Speaking

Name ______________________________ Date ____________________

For each group presentation, mark a tally on the chart every time you hear presenters use one of the target vocabulary words.

Add any words to the chart you hear that are important in the presentations.

Target Word(s)	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Approximate						
Convert						
Cube/Cubic						
Diameter						
Estimate						
Feet						
Formula						
Height						
Inches						
Length						
Measure						
Measurement						
Radius						
Rectangular prism						
Round						
Units						
Volume						
Width						

Creating Equations

Level: High School

Version 8.11.13

Table of Contents

These Language of Mathematics tasks were designed to support students in learning to read word problems and talk about the mathematics. They are accompanied by suggestions for classroom use.

Name ______________________________

Creating Equations

1. If $v = 12R/(r + R)$ write an expression for R in terms of the other variables

2a. Jane, Maria, and Ben each have a collection of marbles. Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles. Find how many marbles Maria has.

2b. Dave sold 40 tickets for a concert. He sold x tickets at \$2 each and y tickets at \$3 each. He collected \$88.

Write two equations connecting x and y.

Solve these two equations to find how many of each kind of ticket he sold.

2c. A rectangle has length of $(x + 5)$ cm and width $(x - 2)$ cm. Its area is 60 cm^2.

Write a quadratic equation, and solve it to find the length and width of this rectangle.

Creating Equations | Annotations

Core Idea

This task affords students opportunities to use algebra in different ways: manipulating a given equation, writing equations to represent situations, solving equations, and interpreting solutions of equations in the situations represented.

Common Core State Standards for Mathematical Content
http://www.corestandards.org/Math/Content/HSA/CED

HIgh School, Algebra. Creating Equations (p. 65)

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities. . . .

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*

HIgh School, Algebra. Reasoning with Equations and Inequalities (p. 65)

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

4. Solve quadratic equations in one variable.

   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. . . .

Common Core State Standards for Mathematical Practice
http://www.corestandards.org/Math/Practice

SMP.1. Make sense of problems and persevere in solving them.	. . . Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships. . . . They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. . . .
SMP.2. Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. . . .
SMP.3. Construct viable arguments and critique the reasoning of others.	Mathematically proficient students . . . justify their conclusions, communicate them to others, and respond to the arguments of others. . . .
SMP.4. Model with mathematics.	Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. . . . They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams . . . and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
SMP.6. Attend to precision.	Mathematically proficient students try to communicate precisely to others. . . . They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. . . .

Common Core State Standards for ELA/Literacy
http://www.corestandards.org/ELA-Literacy

Writing Standards, Grade 9–10 (p. 45)	2. Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
Speaking and Listening Standards, Grades 9–10 (p. 50)	1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 9–10 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
Reading Standards for Literacy in Science and Technical Subjects, grade 9–10 (p. 62)	7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
	4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11–12 texts and topics.

Comments

Purpose of the task. Creating Equations provides opportunities for students to create and use algebraic equations. The task has four questions. Students are given a literal equation with more than two variables (Question 1) and asked to solve for one of the variables, asked to solve a word problem with one variable (Question 2a), and asked to solve word problems that involve simultaneous linear equations in two unknowns (Questions 2b and 2c). Although the different questions are unrelated in context, they are related in mathematical content.

Language of Mathematics tasks. The last section of these annotations provides three Language of Mathematics tasks. These tasks were designed to support students in learning to read word problems and talk or write about their solutions and reasoning. They are:

Jigsaw Reading for Creating Equations Question 2a (pages 14-20)

Reading and Understanding: Creating Equations 1 (pages 21-24)

Mathematically Speaking: Creating Equations 1 (pages 25-29)

Jigsaw Reading provides one way to support students in learning to see the structure of the word problem in Question 2a. The version of Jigsaw Reading provided here fits Question 2a, but the task structure can be revised to fit other word problems with longer texts. It is possible that Jigsaw Reading may work better for word problems that have more than three sentences.

Reading and Understanding: Creating Equations provides a structure for students to work in pairs or small groups. The directions support students as they read and explain each question in this task. The version provided here is designed for the three parts of Question 2, but can be revised to fit Question 1.

Mathematically Speaking provides a structure for students to learn to describe their solutions by first working alone and then working in pairs to describe their solutions both orally and in writing.

These Language of Mathematics tasks are provided as resources to be used, revised, and combined to fit a variety of lesson plans. The overall goals are to minimize direct instruction and introduction by the teacher, and instead provide structure so that the students can grapple with these questions themselves. Students first work alone, then in pairs or groups, and finally in a whole class discussion while always focusing on their mathematical reasoning. This cycle provides ELLs with the opportunity and time to think, practice speaking in pairs or groups, and thus be better prepared to participate in a whole class discussion or a presentation of their reasoning. Students should be encouraged to describe not only *what* they are doing but also, more importantly, *why* they are doing it. Teacher questions and whole class discussions should focus on describing, refining, and comparing students' mathematical reasoning.

Suggestions

Question 1

1. If $v = 12R/(r + R)$ write an expression for R in terms of the other variables

Introducing the question: Whole class and/or small groups

The teacher can introduce the question by:

1. Giving a simple example of an equation using speed, distance, and time or other familiar quantities recently studied.
2. Asking students to discuss the meaning of each question in small groups and to restate the question in their own words. Some possible responses include:
   - In Question 1, *V* is expressed in terms of the variables *r* and *R.* What would the equation look like if *R* was expressed in terms of the variables *V* and *r?*
   - The equation in Question 1 gives *V* in terms of *R* and *r.* Write an equation that gives *R* in terms of *V* and *r.*
   - If *V* is equal to 12*R* divided by the quantity (*r* + *R*)*,* then what would the equation be if we wanted to say *R is equal to* ___________________*?*
   - *If V* = 12*R* ÷ (*r* + *R*)*, then R* = ?

Small group work: Focus students on mathematical reasoning

1. Ask students to work in groups to "undo" the operations.
2. Ask students to describe both orally and in writing:
   - what they did to "undo" each operation.
   - why they did that step.
   - why that step is justified mathematically, asking "What is the mathematical reason for that step?"

After small group work

After students have discussed their solutions in small groups, the teacher can ask one student from each group to write and explain their group's solution on the board along with the reasons for each step. The whole class discussion can focus on any differences in the form of the solutions, the steps taken, or the justifications.

2a. Jane, Maria, and Ben each have a collection of marbles. Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles. Find how many marbles Maria has.

Question 2a

See Jigsaw Reading (pages 14-20) for one way to support students in learning to see the structure of this word problem.

See Reading and Understanding Creating Equations (pages 21-24) for ways to structure the work students do in pairs or small groups.

For Reading and Understanding, students first read the question or problem on their own, then with a partner or in a larger group. Next, students are asked to discuss what the problem is about and what quantities are being measured or counted. Next, students identify and list all numerical values, and, if possible, draw a diagram. Students can use examples with real marbles or other objects to illustrate for each statement several ways in which it might be satisfied.

Introduce the question

1. Ask students to examine the sentence "*Find how many marbles Maria has.*" Then ask students to describe how this sentence may be rephrased. One possible response is as the question, "*How many marbles does Maria have?"*

2. Remind students to state the meaning of the variables they use, e.g., "let *x* be the number of marbles Ben has."

3. Remind students that the variable does not always have to be represented by *x*, it can be represented by any letter.

4. Remind students that they can pick which quantity will be their variable, for example:

    - let *b* be the number of marbles Ben has, or

- let m be the number of marbles Maria has, or
- let j be the number of marbles Jane has.

Small group work: Focus students on mathematical reasoning

Ask students work in groups, ask them to describe both orally and in writing what they did to solve the problem, why they did that step, and why that step is justified mathematically.

After small group work

After students have discussed their solutions in small groups, the teacher can ask one student from each group to write their group's solution on the board along with the reasons for each step. The whole

2b. Dave sold 40 tickets for a concert. He sold x tickets at \$2 each and y tickets at \$3 each. He collected \$88.

Write two equations connecting x and y.

class discussion can focus on any differences in the form of the solutions, the steps taken, or the justifications.

Question 2b

See Reading and Understanding Creating Equations (pages 21-24) for ways to structure the work students do in pairs or small group.

When working on Reading and Understanding, students:

1. Read question 2b on their own.
2. Read question 2b to their partner or to the group.
3. Discuss with each other what the problem is about and what quantity is being measured or counted.
4. Identify and list all numerical values, and, if possible, draw a diagram representing each statement. Students can use examples with objects to illustrate several ways that each statement might be satisfied.

Introduce question 2b

1. Point out the use of "collection of marbles" in the previous question (2a) and the use of "collected \$88" in this question (2b).

2. Ask students how many variables they need for this question.

If students need more help, provide the following guidance:

- Write one equation that shows how many tickets Dave sold.
- Write another equation that shows the amount of money Dave received in ticket sales.
- Use both equations to find how many of each kind of ticket Dave sold.

Small group work: Focus students on mathematical reasoning

As students work in groups, ask them to describe both orally and in writing what they did to solve the problem, why they did that step, and why that step is justified mathematically.

After small group work

After students have discussed their solutions in small groups, the teacher can ask one student from each group to write their group's solution on the board along with the reasons for each step. The whole class discussion can focus on any differences in the form of the solutions, the steps taken, or the justifications.

Question 2c

2c. A rectangle has length of $(x + 5)$ cm and width $(x - 2)$ cm. Its area is 60 cm^2.

Write a quadratic equation, and solve it to find the length and width of this rectangle.

See Reading and Understanding Creating Equations (pages 21-24) for ways to structure the work students do in pairs or small groups.

Introduce question 2c

1. Point out that "cm" means "centimeters."
2. Ask students to list all measurements mentioned in the problem.
3. Ask students to replace a pronoun with a noun by replacing "its" with "the area of the rectangle" and rephrasing the second sentence as "The area of the rectangle is 60 cm^2."

If students need additional guidance:

- Ask students to choose different values for *x* and compute the area of the rectangle in each case to make the computation for area explicit.

- Ask students to draw the figure described in the problem and to label the sides of the figure as *length* and *width*.
- Remind students to state what variables represent.
- Ask students if they remember any formulas associated with a rectangle. If they don't remember any formulas, ask them for the formula that gives the area of a rectangle.
- Clarify, explain, or rephrase the last sentence as "Write an equation showing how the product of the length and width for this rectangle gives you the area."
- Ask students whether the equation is a linear equation or a quadratic equation? Some students may need to expand the product before they can answer this question.
- Ask students "Why is it a quadratic equation?" Some students may need to expand the product before they can answer this.

Small group work: Focus students on mathematical reasoning

As students work in groups, ask them to describe both orally and in writing what they did to solve the problem, why they did that step, and why that step is justified mathematically.

Ask students to:

- Solve the equation and use the solution to find the length and width of the rectangle.
- Check if the length and width found produces a rectangle with area 60 cm^2.
- Discuss why there are two solutions to the quadratic equation and why the negative solution does not yield a rectangle, thus does not satisfy the problem specifications.

After small group work

After students have discussed their solutions in small groups, the teacher can ask one student from each group to write their group's solution on the board and describe the reasons for each step. The whole class discussion can focus on any differences in the form of the solutions, the steps taken, or the justifications.

Jigsaw Reading for question 2a

Adapted from the ELA unit (Walqui, Kolesch, & Schmida, 2012).

Note: Jigsaw Reading is an activity in a very early draft form and has not yet been piloted in classrooms. It may turn out to work best with longer complex mathematics text.

Purpose

This Language of Mathematics task is intended to support students in learning to read and understand word problems. It asks students to think about how the sentences in a word problem are organized, how the text for a word problem flows, and how word problems are predictable. For example, the structure of a word problem typically begins with the statement of some given information, then more information, ending with a question or request for a solution or missing information. The text often includes words that are organizational markers, such as “find,” “solve,” or “write,” or blanks that indicate that an answer is expected. The organization of paragraphs or sentences often follows the sequence: “given information,” “more information,” “request for solution.”

The task requires that students carefully read parts of a word problem to determine where a particular sentence, written on a strip of paper, fits in relation to the other passages. In the process, students begin to focus, without prompting, on how selected grammatical and mathematical terms create cohesion and meaning within and across sentences, and how larger units of text are connected to create a word problem. The goal is to apprentice students into the type of close reading needed to understand more complex word problems and other mathematics texts.

Group structure and materials

- Groups of 3 students
- Copies of the text strips (one complete set of text strips for each group)
- Scissors
- Envelopes (one envelope for each group of students)
- A mathematics task or text that is no longer than a half page (in this case, Creating Equations Question 2b).

Initially, the text on the strips should contain clear organizational markers that are typical of the particular type of mathematics problem it illustrates. As students become more sophisticated readers of mathematics problems, they may benefit from reading and reassembling texts that are clearly organized but do not use understood markers to signal organization.

Structure of the activity

Initially, the teacher describes the overall purpose of the activity by explaining that writers use language to connect ideas within and across sentences and paragraphs in a word problem. In this activity, students will reassemble the text of a word problem whose sentences appear out of order on strips of paper to help students understand how sentences or paragraphs within a word problem are connected with each other. The teacher might introduce this activity with a mathematics task or text that is familiar to the class.

The text for Question 2a is partitioned into three strips (one set), with one sentence on each strip. A complete set of the three strips is placed in each envelope. The teacher distributes and reviews the directions on the student handout and gives each group an envelope of strips.

Process outline

Small group work

1. Describe to the class the directions below. Have students form groups of three.
2. One student distributes the text strips randomly to his or her group members.
3. Each student then reads his or her strip silently and decides where the text fits in the whole word problem: Is it the beginning? The middle? The end? Students must give reasons for their decisions.
4. When everyone in a group appears to be ready, the student who thinks he or she has the first text strip says, "I think I have the first strip because . . ." and, without reading the text aloud, explains their rationale. If any other group members think they have the first strip, they too must explain, "I think I have the first strip because . . ."

5. Once the group decides which student has the first strip, that student reads his or her strip aloud.

6. After hearing the text, the group decides whether it is indeed the first part of the problem. Once agreement is reached, the strip goes face up on the table where group members can refer to it as needed.

7. Students follow the same procedure to reconstruct the rest of the problem, strip by strip.

8. If students feel they have made a mistake along the way, they go back and repair it before continuing.

9. Once the whole process is finished, all group members review the "jig-sawed" text to make sure it has been assembled correctly.

Whole group discussion. The teacher can facilitate a whole group discussion asking students to explain which words, phrases, connectors, or other linguistic features helped them ascertain the order of the strips. The teacher can use strips of transparencies on the overhead (or document camera), or paper strips on a white board (or document camera) to manipulate during student explanations. When warranted, the teacher can provide different ways to state relationships given in the text for the word problem.

Role of the teacher

The overall goal is to minimize direct instruction and introduction by the teacher, and instead provide a structure that allows students to grapple with the organization of a word problem by themselves.

The structure of the activity is that students first work alone, then in pairs or small groups, and finally in a whole class discussion while always focusing on their mathematical reasoning. This cycle provides ELLs with the opportunity and time to think, practice speaking in pairs or groups, and thus be better prepared to participate in a whole class discussion or a presentation of their reasoning.

Jigsaw Reading for Creating Equations Question 2a

2a.	Jane, Maria, and Ben each have a collection of marbles. Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles. Find how many marbles Maria has.

Each sentence of question 2a appears on one of three strips below. Cut along the dotted lines and place each collection of three strips in envelopes in a random order.

Jane, Maria, and Ben each have a collection of marbles.

Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles.

Find how many marbles Maria has.

Jane, Maria, and Ben each have a collection of marbles.

Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles.

Find how many marbles Maria has.

Jane, Maria, and Ben each have a collection of marbles.

Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles.

Find how many marbles Maria has.

Jane, Maria, and Ben each have a collection of marbles.

Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles.

Find how many marbles Maria has.

Jane, Maria, and Ben each have a collection of marbles.

Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles.

Find how many marbles Maria has.

Jane, Maria, and Ben each have a collection of marbles.

Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles.

Find how many marbles Maria has.

Name ____

Jigsaw Reading | Creating Equations Question 2a

Step 1. One person in your group distributes one strip of paper to each person in your group. Read your section silently and decide if your strip appears in the beginning, middle, or end. Use the box below to write your decision and the reason for your decision.

Write beginning, middle, or end in the blank for your strip and finish the sentence below.

I think I have the ___________________ strip because _______________________

____.

Order	Reason for my decision

Step 2. Check to see if everyone is ready and begin.

1. The person who believes they have the beginning starts by saying, "I think I have the beginning because . . . "

2. If more than one person believes they have the beginning, the group must also hear their reasons. Talk within your group to reach agreement. Once your group agrees which person has the beginning, that person reads their text aloud and the strip goes face up on the table.

3. The person who believes they have the middle goes next by saying, "I think I have the middle because . . . "

4. If more than one person believes they have the middle, the group must hear their reasons. Talk within your group to reach an agreement. Once your group agrees which person has the middle, that person reads their text aloud and the strip goes face up on the table.

5. The person who believes they have the end says, "I think I have the end because . . . "

6. Once your group reaches agreement on the order, one person reads the entire problem aloud to the group.

7. Write the entire problem in the space below.

Step 3. Decide whether each strip has information that is known, information that is not known, and/or information that you want to know. In the space where the entire problem is written, write K, N, or W, on top of each part of the beginning, middle, and end strips to indicate the following:

K is for information that is Known.

N is for information that is Not known.

W is for information that you Want to know.

Organize these types of information with your group in the table below.

Information that is known:	
Information that is not known, but is needed to solve the problem:	
Information that you want to know to solve to solve the problem:	

Reading and Understanding Creating Equations

Adapted from work developed by Harold Asturias

Purpose

The purpose of this Language of Mathematics task is to support students in approaching a mathematics problem. It gives students tools for learning to read, understand, and extract relevant information from a problem, and gives them practice using this information to make inroads toward the additional information they are trying to find in order to solve the problem.

Required for use

- Reading and Understanding Creating Equations handout: three copies of the handout for each pair of students.
- Questions 2a, 2b, and 2c: one copy of the worksheet for each student.

Structure of the activity

1. Students begin by individually reading or attempting to read the problem (one of questions 2a, 2b, or 2c), then immediately form pairs to talk through the problem using the Reading and Understanding handout.
2. There are four steps in talking through the problem together, three of which begin with reading the problem aloud:

   Step 1 involves identifying what the problem is about (marbles; concert tickets; a rectangle).

   Step 2 asks students to make explicit what information they are supposed to find (a number of marbles; amounts of two kinds of tickets, each with a different price; the length and width of a rectangle in given units of measurement). Students answer these questions together, both verbally and in writing.

   Step 3 involves a scaffolded set of questions leading to the development of a diagram that represents both the known and unknown information about the quantities in the situation.

   Step 4 asks students to try to act out the problem using real objects to represent the quantities in the situation.

3. Pairs of students should present their diagrams to the class. As they view and interpret other pairs' diagrams, they add details or labels to their own diagrams.

Process outline

1. Students work individually on the problem.
2. Students form pairs and each pair shares one copy of the handout.
3. Pairs of students talk together to answer the questions in Steps 1–3 on the handout in writing.
4. Finally students try to act out the problem using physical objects to represent the quantities in the situation.

Role of the teacher

The overall goal is to minimize direct instruction and introduction by the teacher, and instead provide structure so that the students can grapple with the information and the meaning of the problem themselves.

The structure of the task is that students first work alone, then in pairs or small groups, and finally in a whole class discussion while always focusing on their mathematical reasoning. This cycle provides ELLs with the opportunity and time to think, practice speaking in pairs or small groups, and thus be better prepared to participate in a whole class discussion or a presentation of their reasoning.

Name ______________________________

Reading and Understanding Creating Equations: Questions 2a, 2b, 2c

Name of the problem ____________ (Question 2a, Question 2b, or Question 2c)

Step 1. Partner 1 reads the problem out loud to Partner 2. Answer the question together in writing.

What's the problem about?

Step 2. Partner 2 reads the problem out loud to Partner 1. Answer the questions together in writing.

What is the question in the problem? What are you looking for?

Step 3. Read the problem aloud a third time together. Talk to your partner and answer these questions together in writing.

a. In mathematics problems, we often count or measure things. What are we counting or measuring in this problem?

b. What do you know about the quantities in this problem?

c. What unknown information do you need to find in order to solve the problem?

d. What operations and/or formulas are useful in this problem?

e. Draw a diagram of the problem and label all the information you know. Then try to represent all of the unknown information in your diagram.

Step 4. Use physical objects to represent the quantities in the situation (for example, number of marbles or tickets) and try to act the problem out using the objects.

Mathematically Speaking: Creating Equations

Adapted from R. Santa Cruz (2012) for the Understanding Language Project

Note

Students can first work on the Reading and Understanding Creating Equations task first to ensure that all are ready to begin working on each question in Creating Equations.

Purpose

The purpose of this Mathematically Speaking task is to provide students an opportunity to use important vocabulary orally and in writing *during* or *after* working on a mathematics task. It is intended as a *vocabulary review* and is not meant to be used to preview vocabulary. This task gives students the opportunity to first work on solving a mathematics problem, then to use targeted vocabulary to explain to a partner how they arrived at their solution; moreover, it gives students practice tracking and interpreting vocabulary used by their peers. It is crucial that students do this vocabulary work *after* they solve a mathematics problem that grounds the meanings for words. It is important to recognize that students will use everyday words in their talk while solving the mathematics problem and should not be corrected. Instead the teacher can provide more formal mathematical terms later during a whole class discussion.

Note that developing academic language is more than just learning the target or specialized vocabulary of a problem, lesson, unit, or chapter. For example, comparative structures such as “twice as many” or “3 less than” are syntactic structures that students need to understand and use not separated from solving a math problem, but at the same time as they are working on solving a math problem, so that use the target vocabulary for the purpose of communicating their reasoning.

Required for use

1. For each student: One or more mathematics tasks with solution strategies that require several sentences to describe and explain. (In this case, Creating Equations is used.)

2. For each pair of students: Mathematically Speaking tally charts with target vocabulary words. (In this case, target vocabulary is from Creating Equations.)

Structure of the activity

1. Students first work individually on the four questions in Creating Equations, with the requirement that they begin every problem, even if they are unable to get very far. They record their work on the handout, using additional space as needed, in preparation for working with a partner.

2. Student pairs are formed, and students work together to solve each problem, sharing their individual notes and each making new notes as they work.

3. Each pair then gets one copy of the tally sheet with target vocabulary words. One student explains his or her solution strategy for question 1 to the other student. The listener marks the tally sheet each time a target word is used in the explanation. If a target word is not used in the speaking student's initial explanation, the listening student can ask questions that use the target word(s) for further explanation. (For example, for question 1, target word *equivalent*: "What is equivalent to what in the given equation?" or "What is equivalent to what in your new equation?" For question 2b, target word s*olve*: "How did you know when you were ready to solve your equation?" or "How did you know you had finished solving your equation?") The listening students should encourage the speaking students to keep talking until all target words on the list have been used.

4. Students move on to the next question, switching roles as speaker and listener.

5. The pairs can add words to the tally sheet that come up in their explanations that they think are important or challenging, then share these with the class at the end of the activity.

Process outline

1. Each student receives a copy of the first handout and uses that to show their work on the problems.

2. Students form pairs to finish solving all problems together.

3. Each pair shares a copy of the tally sheet, tallying uses of target vocabulary words for each explanation, and adding words as needed.

Role of the teacher

The overall goal is to minimize direct instruction and introduction by the teacher, and instead provide structure so that the students can grapple with the information and the meaning of the problem themselves.

The structure of the task is that students first work alone, then in pairs or small groups, and finally in a whole class discussion while always focusing on their mathematical reasoning. This cycle provides ELLs with the opportunity and time to think, practice speaking in pairs or small groups, and thus be better prepared to participate in a whole class discussion or a presentation of their reasoning.

The teacher poses questions, takes notes, observes students, and asks students if they have questions or works with small groups with differentiated teaching.

Mathematically Speaking: Individual Work

Name: ___________________________

Solve each question part. Show all your work and use more space on the back page or additional paper if needed.

1. If $v = 12R/(r + R)$ write an expression for R in terms of the other variables	2a. Jane, Maria, and Ben each have a collection of marbles. Jane has 15 more marbles than Ben, and Maria has 2 times as many marbles as Ben. All together they have 95 marbles. Find how many marbles Maria has.
2b. Dave sold 40 tickets for a concert. He sold x tickets at \$2 each and y tickets at \$3 each. He collected \$88. Write two equations connecting x and y.	2c. A rectangle has length of $(x + 5)$ cm and width $(x - 2)$ cm. Its area is 60 cm^2. Write a quadratic equation, and solve it to find the length and width of this rectangle.

Mathematically Speaking: Pair Work

Partner Names: ____________________ & ____________________

Task Name: Creating Equations

1. Partner 1 writes his or her name in the chart and explains solution strategy for Question 1.
2. Partner 2 listens to the explanation and marks a tally on the sheet each time Partner 1 uses a target word.
3. Partner 2 writes his or her name in the chart and explains solution strategy for Problem 2a while Partner 1 marks tallies on the sheet.
4. Switch roles again for Problem 2b, and again for Problem 2c.
5. Both partners add words to the chart that are important in the explanations.

Question 1. Target Words	**Name** ____________	**Question 2a. Target Words**	**Name** ____________
equation		equation	
variable		unknown	
equivalent		known	
expression		solve	
in terms of		solution	
		expression	
		more than	
		two times as many	

Question 2b. Target Words	**Name** ____________	**Question 2a. Target Words**	**Name** ____________
equation		equation	
unknown		unknown	
known		known	
solve		solve	
solution		solution	
expression		expression	
coefficient		length	
sum		width	
		factor	
		binomial	
		quadratic	

Sidewalk Patterns

Level: High School

Version 8.11.13

This task gives students an opportunity to:

- Work with expressions, equations, and functions

Table of Contents

These Language of Mathematics tasks were designed to support students in learning to read word problems and talk about the mathematics. They are accompanied by suggestions for classroom use.

Name ______________________________

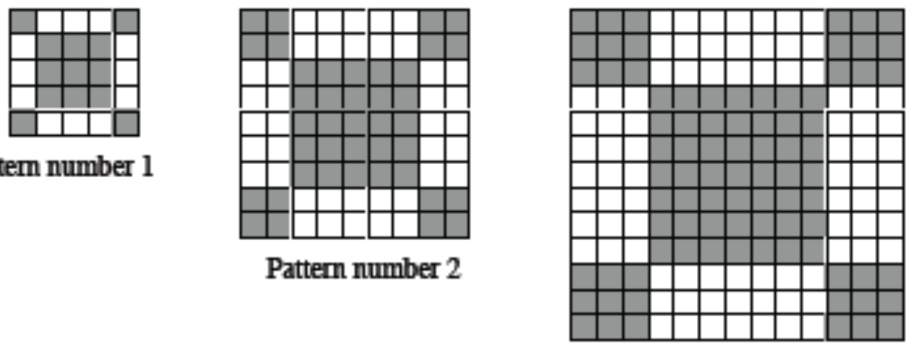
Sidewalk Patterns

This problem gives you the chance to:
• work with expressions, equations, and functions.

In Prague, some sidewalks are made of small square blocks of stone.

The blocks are in different shades to make patterns of various sizes.

This problem is about how the patterns below are related.



Pattern number 1 Pattern number 2 Pattern number 3

1. How can Pattern 1 be changed to make Pattern 2?

   How can Pattern 2 be changed to make Pattern 3?

   Use words, diagrams, or symbols to describe these changes.

Adapted from the Noyce Foundation's Sidewalk Patterns.

2. Complete the table below by writing an arithmetic expression for each number of blocks. Some examples are in the table.

	Pattern 1	Pattern 2	Pattern 3
Number of white blocks	4×3	40	
Number of gray blocks	$(1 + 1 + 1 + 1) + 3^2$		
Total number of blocks	25		

3. In the completed table, there are *three* different arithmetic expressions for the number of white blocks in Patterns 1, 2, and 3.

   Write *one* symbolic expression that represents the number of white blocks in each pattern.

4. In the completed table, there are *three* different arithmetic expressions for the number of gray blocks in Patterns 1, 2, and 3.

   Write *one* symbolic expression that represents the number of gray blocks in each pattern.

5. In the completed table, there are *three* different arithmetic expressions for the total number of blocks in Patterns 1, 2, and 3.

   Write *one* symbolic expression that represents the total number of blocks in each pattern.

Sidewalk Patterns | Annotations

Core Ideas

Sidewalk Patterns presents students with a sequence of three visual patterns created using white and gray blocks, and asks them to write one symbolic expression for the number of each type of blocks. It provides opportunities for work related to the standards listed below.

Note that a finite sequence does not determine exactly one pattern without additional constraints. Because of this, the Common Core State Standards do not require students to infer or guess a single underlying rule for a pattern when given a finite sequence.

Common Core State Standards for Mathematical Content

http://www.corestandards.org/Math/Content/HSA/CED

High School, Functions, Building Functions (p. 70)

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, and use them to model situations and translate between the two forms.

High School, Algebra, Seeing Structure in Expressions (p. 64)

Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

   a. Interpret parts of an expression, such as terms, factors, and coefficients.

   b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Common Core State Standards for Mathematical Practice
http://www.corestandards.org/Math/Practice

SMP.1. Make sense of problems and persevere in solving them.

. . . Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. . . . They can understand the approaches of others to solving complex problems and identify correspondences between different

SMP.3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students . . . are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. . . .

SMP.4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. . . . They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

SMP.6. Attend to precision.

Mathematically proficient students look closely to discern a pattern or structure. . . . They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. . . .

SMP.7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. . . . They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. . . .

SMP.8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. . . .

Common Core State Standards for ELA/Literacy
http://www.corestandards.org/ELA-Literacy

Grades 9–10, Writing (p. 45)	1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence. 2. Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content. (Use precise language and domain-specific vocabulary to manage the complexity of the topic.)
Grades 9–10, Speaking and Listening, Comprehension and Collaboration (p. 50)	1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 9–10 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively. 2. Integrate multiple sources of information presented in diverse media or formats (e.g., visually, quantitatively, orally), evaluating the credibility and accuracy of each source.
Grades 9–10, Reading for Informational Text, Craft and Structure (p. 62)	4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9–10 texts and topics.
Grades 9–10, Reading for Informational Text, Integration of Knowledge and Ideas (p. 62)	7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

Comments

Purpose of the task. This task presents substantial opportunities for ELLs to directly engage in the mathematics, present their thinking about the mathematics, develop verbal explanations of their solutions, and develop explanations of their solutions in writing.

The nature of the task invites a wide range of approaches, strategies, and representations to come into play as students move from cases to generalizations, and from visual to numerical to algebraic expressions. This range presents a dynamic set of resources for ELL students to connect their own thinking to the language used by their peers, and for all students to build deeper and clearer understandings of the mathematics.

When working with ELLs, focus on supporting verbal and written communication, including creating space within the task for students to refine explanations of their strategies and generalizations.

Structure for framing the task. Part of the work of the task for all students must be to make connections among distinct approaches, strategies, and representations, including:

- identifying algebraic expressions that are equivalent.
- developing and clarifying approaches that are partial or incomplete.
- making generalizations from cases.
- identifying strategies that appear distinct but are algebraically equivalent.

Using the variety of ideas generated by this task as a resource for all students would support SMP.1 ("Make sense of problems and persevere in solving them"), SMP.7 ("Look for and make use of structure"), SMP.8 ("Look for and express regularity in repeated reasoning"), and SMP.3 ("Construct viable arguments and critique the reasoning of others").

Language of Mathematics tasks. The last section of this document provides handouts and teaching suggestions for two Language of Mathematics tasks. These were designed to support students in learning to read word problems and talk about their solutions and reasoning. The tasks are:

Reading and Understanding Sidewalk Patterns (pages 17–18). This provides a structure for students to work in pairs or small groups. The directions support students as they read and explain each question in this task.

Mathematically Speaking: Sidewalk Patterns Poster Presentation (page 19). This provides a structure for students to describe their solutions both orally and in writing (available on the Understanding Language web site).

These Language of Mathematics tasks are provided as resources to be used, revised, and combined to fit a variety of lesson plans. The overall goals are to minimize direct instruction and introduction by the teacher, and instead provide structure so that the students can grapple with the questions themselves. Students first work alone, then in pairs or small groups, and finally in a whole class discussion while always focusing on their mathematical reasoning. This cycle provides ELLs with the opportunity and time to think, practice speaking in pairs or groups, and thus be better prepared to participate in a whole class discussion or a presentation of their reasoning. Students should be encouraged to describe not only *what* they are doing but also, more importantly, *why* they are doing it. Teacher questions and whole class discussions should focus on describing, refining, and comparing students' mathematical reasoning.

Focus students on mathematical reasoning. As students work in groups, when they make presentations, and during whole class discussions, ask them to describe both orally and in writing:

- What they did to solve a problem or find an answer
- Why they did that step, and
- Why that step is justified mathematically; for example, asking "What is the mathematical reason for that step?"

Suggestions

Task as a whole. A likely hurdle for ELLs (and other students) is identifying and naming salient quantities: total number of blocks, number of gray blocks, number of white blocks, and possibly others.

- Language development in mathematics requires students to use language in talking, listening, reading, and writing. This is where paired work is essential so each student talks and is listened to.
- While working with pairs or individual students directly, refer to strategies used by other pairs or groups of students.
- Allow a variety of approaches to take shape. then stop pair conversations to have the whole class share out.

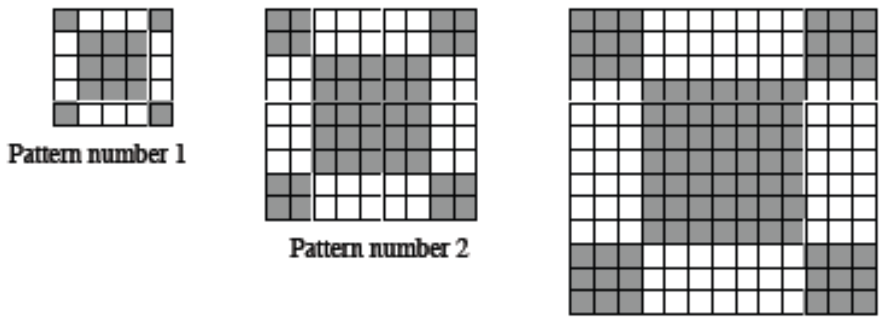
Before Question 1

- Before students work on Question 1, ask them to work alone, look at the first two patterns and write down responses to these questions: "What is the same for both patterns? How are the two patterns different?" Then ask students to share their responses with a partner.
- After the groups work on the question, have a few groups share their findings. The variety of oral explanations will create opportunities for connections between oral and written descriptions.

In Prague, some sidewalks are made of small square blocks of stone.

The blocks are in different shades to make patterns of various sizes.

This problem is about how the patterns below are related.



Pattern number 3

1. How can Pattern 1 be changed to make Pattern 2?

   How can Pattern 2 be changed to make Pattern 3?

   Use words, diagrams, or symbols to describe these changes.

Question 1

Students may move too quickly without understanding each other's reasoning or developing their own reasoning.

- Ask students to fill in the table together in pairs. While working in pairs, they should check each other's work and agree before going on.

- Go over the table as a class and make sure everyone agrees on the values of the expressions, before they go on. This is an opportunity for students to identify equivalent and non-equivalent arithmetic expressions before they use algebraic expressions for Question 2. It is also an opportunity for students to explain correspondences between terms in arithmetic expressions and areas in diagrams. For example, in Pattern 1 the 3^2 ("3 squared") in the table represents the area of a 3 by 3 square and each of the four 3s represents the area of a 1 by 3 rectangle.

Question 2

2. Complete the table below by writing an arithmetic expression for each number of blocks. Some examples are in the table.

	Pattern 1	Pattern 2	Pattern 3
Number of white blocks	4×3	40	
Number of gray blocks	$(1 + 1 + 1 + 1) + 3^2$		
Total number of blocks	25		

The question may need clarification.

Ask pairs to:

- Make an observation in words (verbally) to their partner about how the number of white blocks in Pattern 1 compares with the number of white blocks in Patterns 2 and 3.
- Write down what their partner said.
- Read what they wrote to their partner, and vice versa.
- Refine or synthesize each observation as needed.

Have one partner to express the relationship between the numbers of white blocks in the patterns algebraically, with expressions using letters, then have the other partner translate the expressions into words. Next, the pair revises the expressions as needed.

Allow a variety of approaches to take shape, then stop pair conversations to have the whole class share out. Agree on one or two approaches, e.g., using *n* to represent the number of the pattern and using *s* for the side length of any pattern.

Review as a class several examples of what students are expressing about what they notice. This will give everyone a chance to "see and hear" the reasoning used by their peers and to make their constructions visible on the board on a larger scale.

This process can repeated after students work on Question 4 and again after Question 5.

Language of Mathematics tasks

Two Language of Mathematics tasks designed to work together

- Reading and Understanding Sidewalk Patterns
- Mathematically Speaking: Sidewalk Patterns Poster Presentation.

Reading and Understanding handout adapted from handout developed by Harold Asturias.

Mathematically Speaking handout adapted from the work of R. Santa Cruz (2012) for the Understanding Language Project.

Purpose

Reading and Understanding Sidewalk Patterns. The purpose of this task is to support students in learning to read, understand, and extract relevant information from a mathematics problem and, in doing so, to develop an approach to solving the problem.

Mathematically Speaking: Sidewalk Patterns Poster Presentation. The purpose of this task is to support students by providing a structure for them to describe their solutions by first working alone, then working in pairs to describe their solutions both orally and in writing, and finally making a poster presentation. The tally sheet on page 19 is designed to be used during the presentations, but can also be revised to support students as they prepare their presentations.

Required for use

- Reading and Understanding handout: one copy for each pair of students.
- Square tiles or grid paper, poster paper, and markers.
- Mathematically Speaking tally sheet with target vocabulary: one copy for each pair of students.

Structure of the activity

- Students begin by reading or attempting to read the problem individually, and then immediately form pairs to talk through the problem together using the handout provided. There are four steps in talking through the problem together, three of which begin with reading the problem aloud.

- The *first step* involves identifying what the problem is about (e.g., sidewalks made of small square blocks of stone, different shaded blocks, patterns formed using the square blocks).
- The *second step* involves asking what information students are supposed to find (look for patterns to find the number of white blocks and gray blocks for each pattern). Students answer these questions together, both orally and in writing.
- The *third step* involves a scaffolded set of questions leading to descriptions of relationships between the number of the patterns and a given quantity in the pattern: number of white blocks, number of gray blocks, total number of blocks.
- The *fourth step* is for students to make a poster (in groups of four) to show how they arrived at a symbolic expression for the numbers of blocks in each pattern. Students must show that their expression works for each pattern in the sequence, and use multiple representations to explain their thinking (using diagrams, tables, words, and expressions and equations). Poster paper and grid paper are provided.

Before asking students to present to the whole class, provide students an opportunity in the small groups to think about, write, and practice orally how to use the target vocabulary, using the handout "Mathematically Speaking" on page 19.

- Groups present their work to the class using the target vocabulary of the lesson (e.g., *pattern, sequence, term, total number, squaring, expression*). Where necessary, the teacher will pose questions to elicit the appropriate vocabulary for discussing the patterns in the sequence of figures.
- As groups present their work, the class will tally the number of times the target vocabulary of the lesson is used.

Process outline

- Students work individually on the problem.
- Students form pairs and begin to talk through the problem, sharing one copy of the Reading and Understanding handout.
- Pairs of students talk together to answer the questions in Steps 1–3 on the Reading and Understanding handout, both orally and in writing.

- Finally students work in groups of four to make a poster, showing how they arrived at a solution. They explain their thinking using multiple representations: words, expressions and equations, tables, diagrams, first to the group, then to the whole class. During the presentation to the class, listening students tally word use on the Mathematically Speaking handout (page 19).

Reading and Understanding Sidewalk Patterns

Names: ______________________ and ______________________

Step 1. Read the problem out loud to a peer. Try to answer these questions.

What's the problem about?

Step 2. Read the problem again.

What are the questions in the problem? What are you looking for?

Step 3. Read the problem a third time. Talk to your partner about these questions.

a. What do you observe as you go from one pattern to another? What is changing and what is staying the same? Talk about the pattern you see in the white blocks and the gray blocks.

b. How can you find the number of white blocks and gray blocks in each pattern, along with the total number of blocks?

Reading and Understanding Sidewalk Patterns

Names: ______________________ and ______________________

c. What is a relationship between the number of white blocks in each pattern and the number of the pattern (1, 2, or 3)?

d. What is a relationship between the number of gray blocks in each pattern and the number of the pattern (1, 2, or 3)?

e. What is a relationship between the total number of blocks in each pattern and the number of the pattern (1, 2, or 3)?

Step 4. Work in groups of four (4) to make a poster showing how you arrived at a symbolic expression to find the total number of blocks in each pattern. Show that your expression works for each pattern in the sequence and use multiple representations to explain your thinking (diagrams, tables, words, and expressions or equations). Poster paper and grid paper are provided.

Create an oral presentation of your ideas using the target vocabulary on the handout "Mathematically Speaking" (next page). When you present your ideas to the whole class, you will be expected to use these words. Ask the teacher for help if you do not understand how to use the vocabulary.

As your group presents its work, the class will tally the number of times the target vocabulary of the lesson is used. A chart is provided on the handout "Mathematically Speaking" (next page).

Mathematically Speaking: Sidewalk Patterns Poster Presentation

Names: ______________________ and ______________________

For each group's presentation, mark a tally on the chart every time you hear the presenter use one of the target vocabulary words.

Add to the chart any words you hear that are important in the presentations.

Target Words	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Pattern						
Sequence						
Term						
n^{th} term						
Total number						
Squaring						
Squares						
Rectangles						
White blocks						
Gray blocks						
Arithmetic expression						
Symbolic expression						