# CCRABES Math Workshop

PART I

## Participant Packet

ipdae INSTITUTE FOR THE PROFESSIONAL DEVELOPMENT OF ADULT EDUCATORS

# College and Career Readiness Standards for Mathematics Adult Basic Education

## CCRABES Math Workshop

PART I

July 2015

The IPDAE project is supported with funds provided through the Florida Department of Education and Division of Career Adult Education.

# CCRABES Math Workshop

Rod Duckworth, Chancellor
Career and Adult Education, Department of Education

Zelda Rogers, Senior Educational Program Director
Adult Education, Career and Adult Education

Ila Waite-Burns, Program Specialist
Adult Education, Career and Adult Education

Dr. Marta Cronin, Assistant Dean of the School of Education
June Rall, Director of IPDAE
Joseph Reed, Technology Director IPDAE
Tamara Serrano, Project Support Specialist for IPDAE

Bonnie Goonen, National Consultant and Trainer

Ronald Cruz, Lead Developer and Trainer

**Trainers**

Ronald Cruz
Jesika Curry
Dr. Kim Milner
Lynne Ralston

ipdae
INSTITUTE FOR THE PROFESSIONAL DEVELOPMENT OF ADULT EDUCATORS

The IPDAE project is supported with funds provided through the Florida Department of Education and Division of Career Adult Education.

# CCRABES Math Workshop

## Agenda

8:30 a.m. – 10:20 a.m.

Objectives

CCRS What and Why: Domains and Critical Shifts

Major Areas of Focus

The Mirror Has Two Faces: Math Instruction Then and Now

Break 10:20 a.m. – 10:30 a.m.

10:30 a.m. – 12:00 p.m.

Instructional Tools for Problem Solving
(The Rational Number System and Operations and Algebraic Thinking)

A. Number Line
B. Colored Counters
C. Geoboard

Lunch 12:00 p.m. – 1:00 p.m.

1:00 p.m. – 2:15 p.m.

World Innovation Summit for Education (WISE)

Beach Ball Brainstorming

Making Sense of Area Formulas Using Index Cards
(Geometry)

Break 2:15 p.m. – 2:20 p.m.

2:20 p.m. – 3:30 p.m.

Math Icebreaker: Mind Reading

Solving Equations Using Algebra Tiles
(Expressions and Equations)

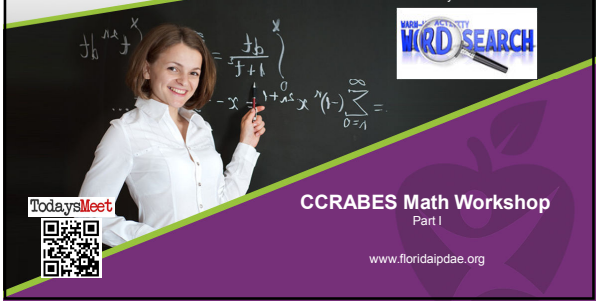
The IPDAE Website

Workshop Wrap-Up

Complete Evaluation Form

ipdae INSTITUTE FOR THE PROFESSIONAL DEVELOPMENT OF ADULT EDUCATORS

Please pick-up a copy of the Word Search activity sheet.




**CCRABES Math Workshop**

Part I

www.floridaipdae.org

---

ipdae BY EDUCATORS FOR EDUCATORS

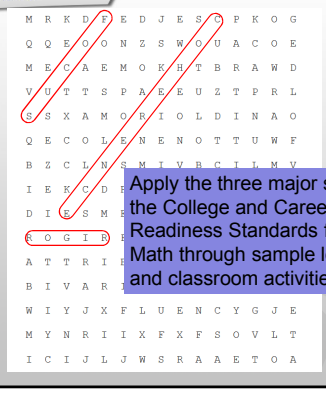
Training Facilitators

# Meet your trainers!

---

ipdae BY EDUCATORS FOR EDUCATORS

Objectives




Apply the three major shifts in the College and Career Readiness Standards for ABE Math through sample lessons and classroom activities.

## Objectives

Recognize and implement the suggested Mathematical Practices into daily classroom instruction.

- Build Solution Pathways and Lines of Reasoning
- Abstracting Problems
- Furthering Lines of Reasoning
- Mathematical Fluency
- Evaluating Reasoning and Solution Pathways

## Objectives

Identify the major domains of the CCR Standards for ABE Math.

1. Base Ten Numbers and Operations
2. Algebraic Thinking
3. Measurement and Data
4. Geometry
5. Fractions and Operations
6. Expressions and Equations
7. The Number System
8. Ratios and Proportional Relationships
9. Statistics and Probability
10. Functions

## Objectives

Unpack the standards and increase fluency in the use of appropriate mathematics vocabulary.

ipdae BY EDUCATORS FOR EDUCATORS

Strategy 1

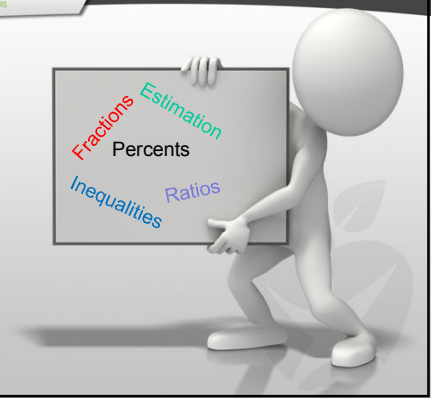
## Silent Discussion

- Assign a different colored marker to each member of the group.
- All participants will not be allowed to talk.
- Members of the group will only communicate by writing on the poster paper provided.
- All members of the group must contribute to the discussion by writing on the poster. Pay attention to what others are writing to prevent duplicating information.
- Your task is: **Write down all the key mathematical skills and concepts within your assigned NRS level.**

ipdae BY EDUCATORS FOR EDUCATORS

## Silent Discussion

**For example:**




ipdae BY EDUCATORS FOR EDUCATORS

## The Mirror Has Two Faces

ipdae BY EDUCATORS FOR EDUCATORS

Strategies for Problem Solving

## Instructional Tools for Problem Solving

- Colored Counters
- Number Line
- Double Number Line
- Geoboard (Rectangular)
- Geoboard (Polar)

ipdae BY EDUCATORS FOR EDUCATORS

CCR ABE Standards

CCR.MA.ABE.2.1.4 Work with addition and subtraction equations.
CCR.MA.ABE.2. 2.3 Represent and solve problems involving multiplication and division.
CCR.MA.ABE.2. 2.4 Understand properties of multiplication and the relationship between multiplication and division.
CCR.MA.ABE.5. 2.1 Develop understanding of fractions as numbers using denominators of 2, 3, 4, 6, or 8.
CCR.MA.ABE.5. 2.2 Develop understanding of equivalent fractions.
CCR.MA.ABE.5. 3.2 Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.
CCR.MA.ABE.5. 3.5 Apply and extend previous understanding of multiplication and division to multiply and divide fractions.
CCR.MA.ABE.7.4.1 Apply and extend previous understandings of numbers to the system of rational numbers.
CCR.MA.ABE.7.4.2 Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

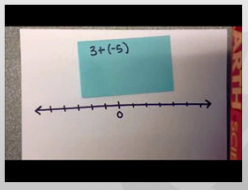
Standards of Mathematical Practice

## Mathematical Practices

- MP.1 Building Solution Pathways and Lines of Reasoning
- MP.2 Abstracting Problems
- MP.3 Furthering Lines of Reasoning
- MP.4 Mathematical Fluency
- MP.5 Evaluating Reasoning and Solution Pathways

Integers, Operations and Equality

## Colored Counters




Strategies for Problem Solving

## Sample Problem

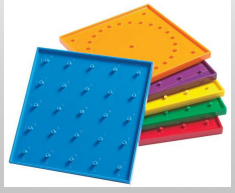
1. Amanda has $4.00 remaining in her checking account right before she made an Internet purchase from Itunes for a discounted music album for $6.00. How much will be the balance on her checking account after her purchase?

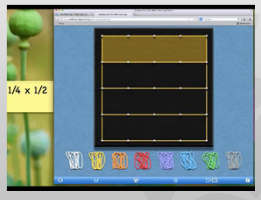
## Sample Problem

2. Jill places 3 apples and 4 oranges inside a brown paper bag. If she pulls a fruit at random from the same bag, what is the probability that she will pull out an apple?

   Jill pulled out an apple from the bag. If she tries to pull out another fruit at random from the same bag, what is the probability that she will pull another apple?

## Multiplication and Division of Fractions using Geoboards

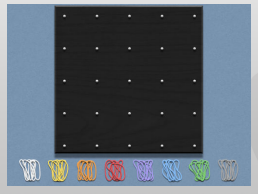






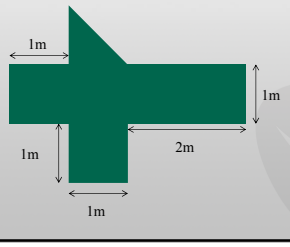

## Sample Problem

3. A cupcake recipe asks for $\frac{3}{4}$ of a cup of butter. Tony wants to make $\frac{1}{2}$ of the original recipe. How many cups of butter will Tony need?



## Sample Problem

4. Kelly wants to cover a section of her garden below with turf. How much turf does she need to cover the entire area?



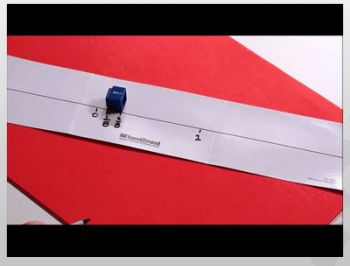



## Sample Problem

5. A recent polling of registered voters showed $\frac{1}{2}$ voting for Democrats, $\frac{1}{3}$ voting for Republicans and the rest were undecided.
   a. What fraction of the polled voters were undecided?
   b. Use a circle graph to represent the 3 groups polled.
   c. What percent of the population were undecided?
   d. If there were 300 registered voters, exactly how many of those voters were undecided?



## Number Line



Download on the App Store

## Sample Problem

6. When building his deck, John needed several decking boards $\frac{1}{3}$ ft in length. How many $\frac{1}{3}$ ft decking boards can he make by cutting a 6ft decking board sold at a local hardware?

   A couple of days later, John needed some decking boards $\frac{2}{3}$ ft in length. How many $\frac{2}{3}$ ft decking boards can he make if he bought the same 6ft decking board sold at the local hardware?



## Sample Problem

7. Strawberries that grow during cool temperatures have firmer fruit than those grown during warm humid weather. In Florida, Fall Season temperatures of 70 to 80 F during the day and 60 to 65 F during the night result in a ripening period of about 30 days.

   A local weather forecast for the winter announced daytime temperatures to drop by 50 F. During this time, at what daytime temperatures will Florida strawberries be growing?



## Sample Problem

8. You are listening to your favorite playlist on your MP3 player. If each song last $3\frac{3}{4}$ minutes, how long will it take to listen to your playlist of 5 songs?

## Group Problem Solving Activity

1. Work together to complete the word problem set assigned to your group using any of the available manipulatives.
2. Select one of the problems you completed to be presented to the entire group.
3. Write the word problem on the poster paper provided to your group.
4. Use the remaining space on your poster to show your group's solution.
5. Be prepared to explain and demonstrate how you solved your problem.
6. Assign members of your group to be presenters.



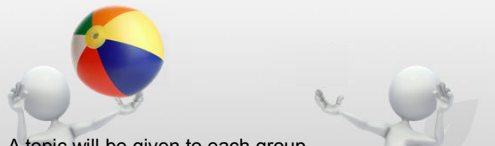
## Group Presentations

1. Groups have 4 minutes to present their work.
2. Presenters must bring their own poster and manipulatives to the front of the room for their presentation.
3. Groups should introduce themselves and read the word problem at the beginning of the presentation.
4. The ELMO may be used to demonstrate solution to the word problems.
5. Prepare to respond to follow-up questions from participants.
6. Groups must take their poster and manipulatives back to their group tables after their presentation.

ipdae BY EDUCATORS FOR EDUCATORS

Strategy 2

## Beach Ball Brainstorming

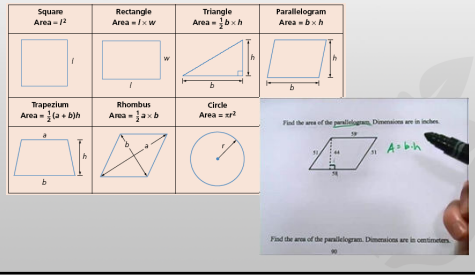


1. A topic will be given to each group.
2. The person holding the ball will call out a word/phrase related to the topic and throw the ball to another member.
3. The person catching the ball will then call out a related word/phrase.
4. The process continues until time is called.

ipdae BY EDUCATORS FOR EDUCATORS

Area Formulas

## Making Sense of Area Formulas







ipdae BY EDUCATORS FOR EDUCATORS

Content Standards

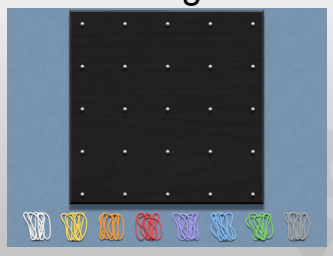
**CCR ABE Standards**

CCR.MA.ABE.3. 2.5 Understand concepts of area measurement and relate area to multiplication and addition.
CCR.MA.ABE.4. 2.2 Reason with shapes and their attributes.
CCR.MA.ABE.2. 3.3 Generate and analyze patterns.
CCR.MA.ABE.4. 4.1 Draw, construct, and describe geometrical figures and describe the relationships between them.
CCR.MA.ABE.4. 3.3 Classify two-dimensional figures into categories based on their properties.
CCR.MA.ABE.4. 3.4 Solve mathematical and real-world problems involving area, surface area, and volume.

**Mathematical Practices**

- MP.1 Building Solution Pathways and Lines of Reasoning
- MP.3 Furthering Lines of Reasoning
- MP.4 Mathematical Fluency
- MP.5 Evaluating Reasoning and Solution Pathways

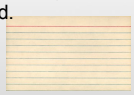
## Areas of Squares and Rectangles





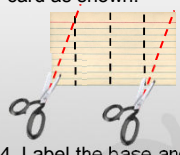
## Area of Parallelograms

1. Take a piece of index card.

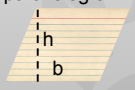


2. Fold it into fourths.

3. Cut both ends of the card as shown.



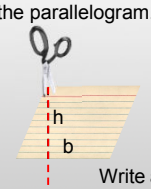
4. Label the base and height of the parallelogram.



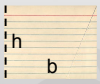



## Area of Parallelograms

5. Cut along the height of the parallelogram.




6. Place the cut piece on the other side forming a rectangle.

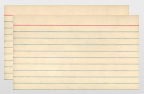



Write an equation for the area of the shape you formed using your writing boards.

**Area** $= bh$

## Area of Triangles

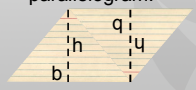
1. Take two index cards.



2. Create two congruent isosceles triangles by folding and cutting.

3. Label the bases and heights of both triangles.

4. Place the two triangles side by side to form a parallelogram.

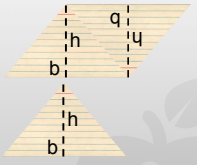



ipdae BY EDUCATORS FOR EDUCATORS
Area Formulas

## Area of Triangles

Write an equation for the area of the resulting figure.

Write an equation for the area of one triangle from the resulting figure.

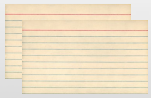



$$\textbf{Area} = \frac{1}{2}bh$$

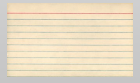
ipdae BY EDUCATORS FOR EDUCATORS
Area Formulas

## Area of Trapezoids

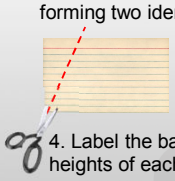
1. Take two index cards.



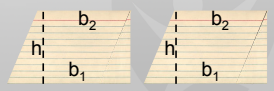
2. Stack them on top of each other.



3. From one corner cut straight forming two identical trapezoids.

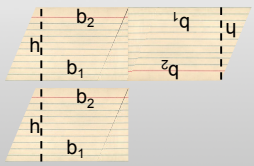


4. Label the bases and heights of each trapezoid.



## Area of Trapezoids

5. Place the trapezoids side by side to form a parallelogram.




Write an expression for the area of the resulting figure.

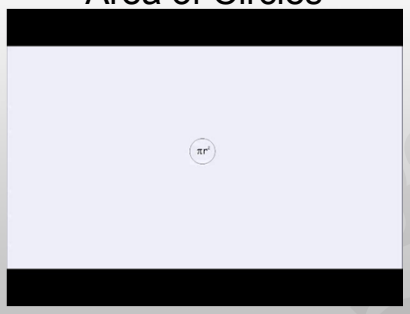
$$(b_1 + b_2)h$$

Write an equation for the area of one trapezoid.

**Area =** $\frac{1}{2}(b_1 + b_2)h$



## Area of Circles



## Content Standards

**CCR ABE Standards**

CCR.MA.ABE.1. 2.3 Use place value understanding and properties of operations to perform multi-digit arithmetic.
CCR.MA.ABE.1. 2.6 Solve problems involving the four operations, and identify and explain patterns in arithmetic.
CCR.MA.ABE.2. 3.3 Generate and analyze patterns.
CCR.MA.ABE.7. 4.1 Apply and extend previous understandings of numbers to the system of rational numbers.

**Mathematical Practices**

- MP.1 Building Solution Pathways and Lines of Reasoning
- MP.2 Abstracting Problems
- MP.4 Mathematical Fluency

By Extending the Strategy:

- MP.3 Furthering Lines of Reasoning (by extending the activity)
- MP.5 Evaluating Reasoning and Solution Pathways

## Abstracting Problems

### Mind Reading

1. Think of a number from 1 – 10.
2. Take that number and multiply it by 9.
3. Take the result and add the digits together (i.e., 72 → 7 + 2 = 9).
4. Take that number and subtract 5.
5. Take that result and equate it to a letter of the alphabet (1 = A, 2 = B, 3 = C, and so on).
6. Think of a country beginning with that letter.
7. Think of an animal that begins with the SECOND letter of the country name.

## Abstracting Problems

### Mind Reading

1. Choose a number from 1 – 10.
2. Find the **product** of that number and 9.
3. Find the **sum** of its digits (i.e., 72 → 7 + 2 = 9).
4. Find 5 **less than** that number.
5. Take that result and equate it to a letter of the alphabet (1 = A, 2 = B, 3 = C, and so on).
6. Think of a country beginning with that letter.
7. Think of an animal that begins with the SECOND letter of the country name.

ipdae BY EDUCATORS FOR EDUCATORS

Expressions and Equations

# Modeling and Solving Equations Using Algebra Tiles

ipdae BY EDUCATORS FOR EDUCATORS

Content Standards

**CCR ABE Standards**

CCR.MA.ABE.2. 1.4 Work with addition and subtraction equations.
CCR.MA.ABE.2. 3.4 Write and interpret numerical expressions.
CCR.MA.ABE.6. 3.1 Apply and extend previous understandings of arithmetic to algebraic expressions.
CCR.MA.ABE.6. 4.1 Use properties of operations to generate equivalent expressions.
CCR.MA.ABE.6. 3.2 Reason about and solve one-variable equations and inequalities.
CCR.MA.ABE.6. 4.2 Solve mathematical and real-life problems using numerical and algebraic expressions and equations.

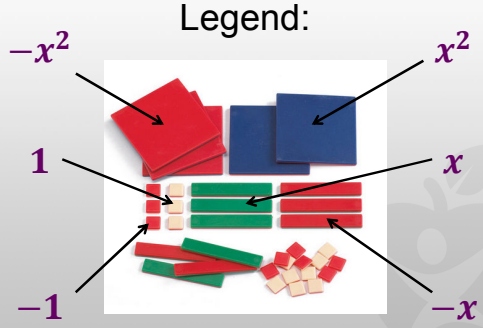
**Mathematical Practices**

- MP.1 Building Solution Pathways and Lines of Reasoning
- MP.2 Abstracting Problems
- MP.3 Furthering Lines of Reasoning
- MP.4 Mathematical Fluency
- MP.5 Evaluating Reasoning and Solution Pathways

ipdae BY EDUCATORS FOR EDUCATORS

Expressions and Equations

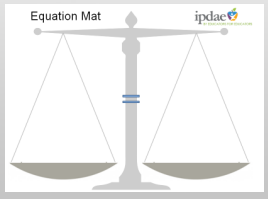
## Legend:



## Expressions and Equations

# Basic Rules:

1. Both sides of the equation must remain balanced.
2. Any pair of opposite signs/charges (or zero pair) cancel out, has no value, and must be taken out of the mat.

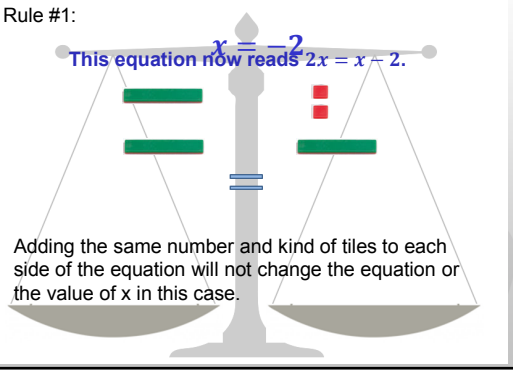


## Modeling Equations

Rule #1:

$x = -2$

This equation now reads $2x = x - 2$.

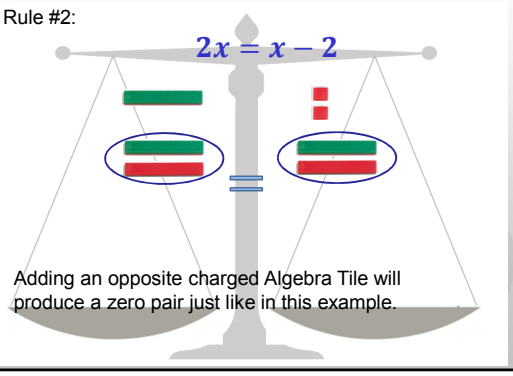


Adding the same number and kind of tiles to each side of the equation will not change the equation or the value of x in this case.

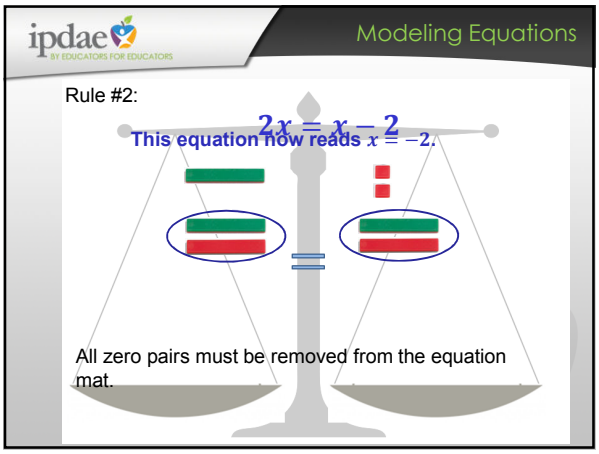
## Modeling Equations

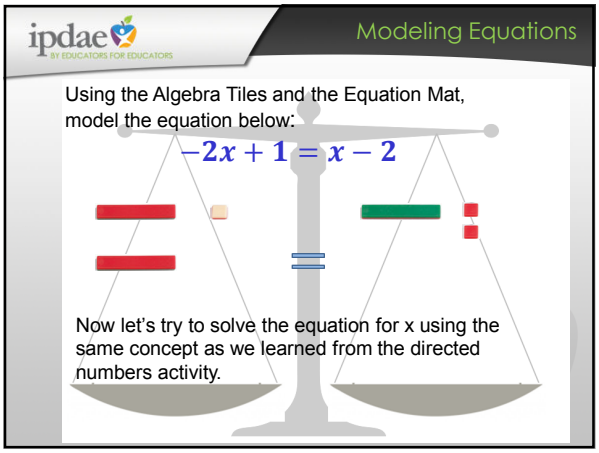
Rule #2:

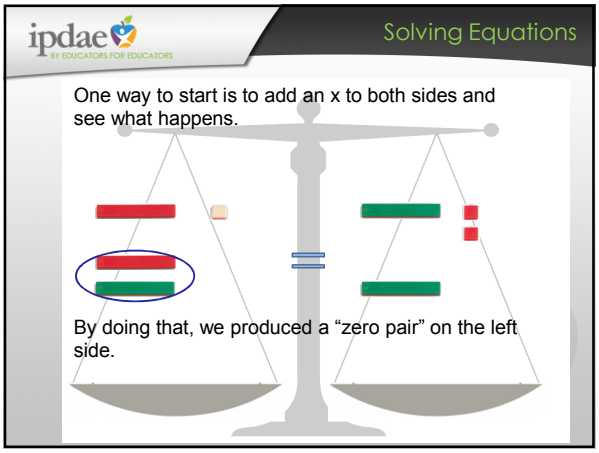
$2x = x - 2$



Adding an opposite charged Algebra Tile will produce a zero pair just like in this example.

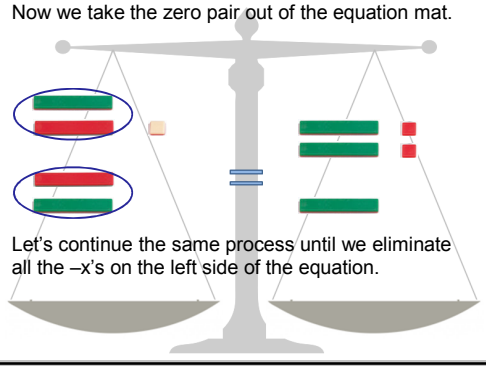

ipdae
BY EDUCATORS FOR EDUCATORS
Modeling Equations
Rule #2:
$2x = x - 2$
This equation now reads $x = -2$.
All zero pairs must be removed from the equation mat.


ipdae
BY EDUCATORS FOR EDUCATORS
Modeling Equations
Using the Algebra Tiles and the Equation Mat, model the equation below:
$-2x + 1 = x - 2$
Now let's try to solve the equation for x using the same concept as we learned from the directed numbers activity.


ipdae
BY EDUCATORS FOR EDUCATORS
Solving Equations
One way to start is to add an x to both sides and see what happens.
By doing that, we produced a "zero pair" on the left side.

## Solving Equations

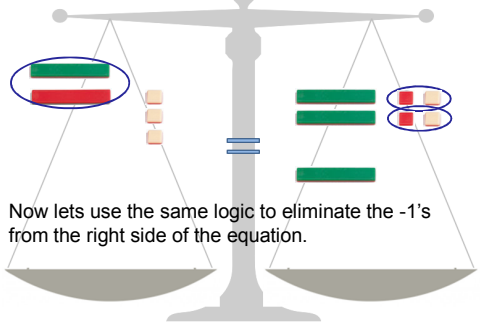
Now we take the zero pair out of the equation mat.



Let's continue the same process until we eliminate all the –x's on the left side of the equation.

## Solving Equations

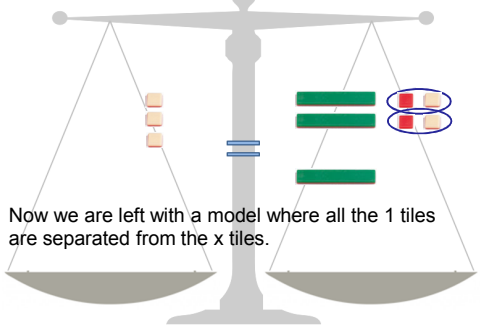
Again, we take the zero pair out of the equation mat.



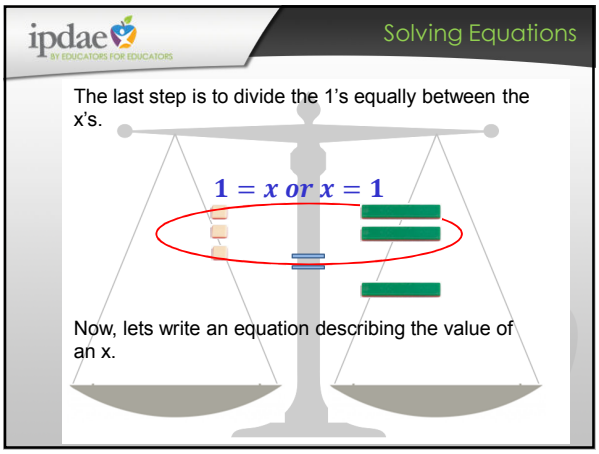
Now lets use the same logic to eliminate the -1's from the right side of the equation.

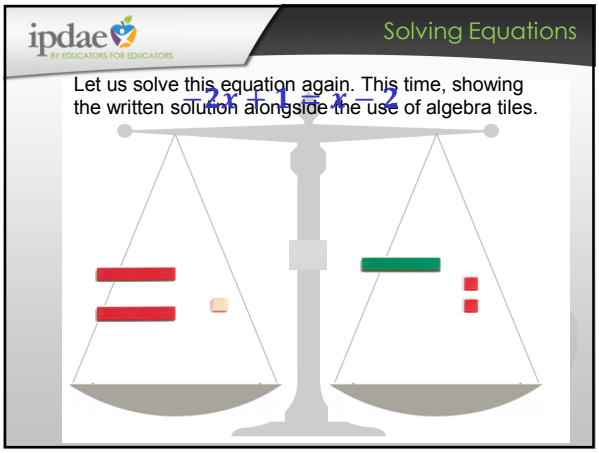
## Solving Equations

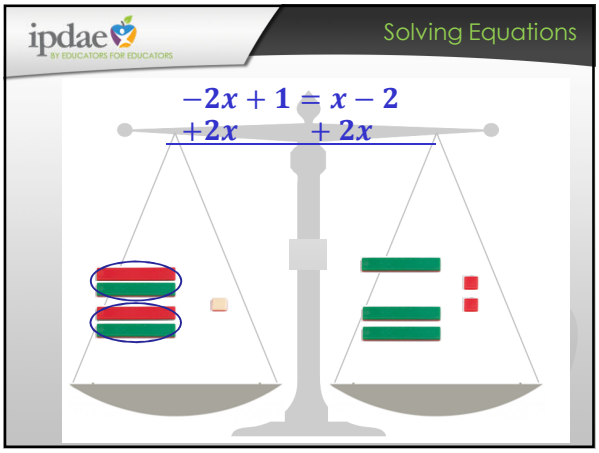
Again, we take the zero pair out of the equation mat.

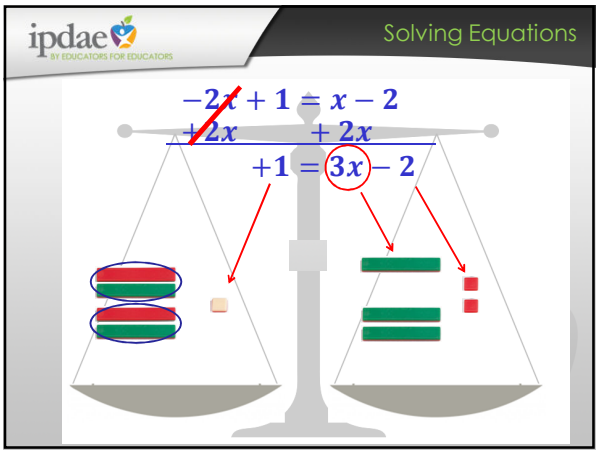


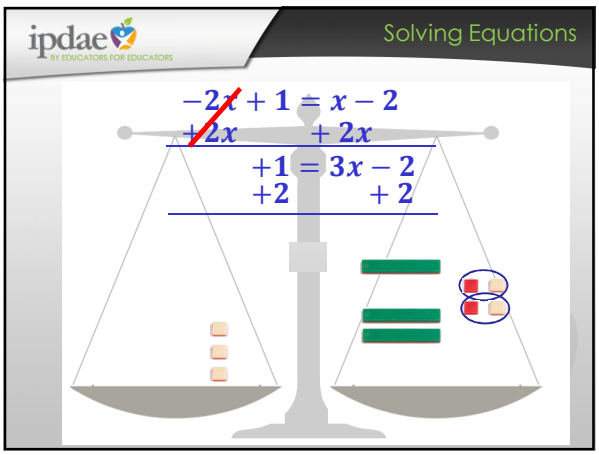
Now we are left with a model where all the 1 tiles are separated from the x tiles.

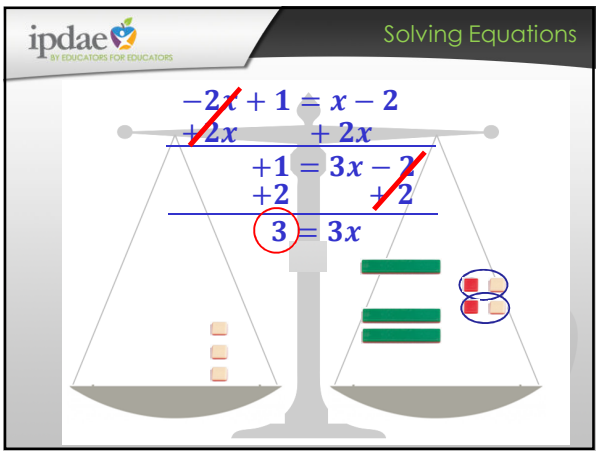

ipdae
BY EDUCATORS FOR EDUCATORS
Solving Equations
The last step is to divide the 1's equally between the x's.
$1 = x \text{ or } x = 1$
Now, lets write an equation describing the value of an x.


ipdae
BY EDUCATORS FOR EDUCATORS
Solving Equations
Let us solve this equation again. This time, showing the written solution alongside the use of algebra tiles.
$-2x + 1 = x - 2$


ipdae
BY EDUCATORS FOR EDUCATORS
Solving Equations
$-2x + 1 = x - 2$
$+2x \qquad + 2x$


ipdae
BY EDUCATORS FOR EDUCATORS
Solving Equations
$-2x + 1 = x - 2$
$+2x \qquad + 2x$
$+1 = 3x - 2$

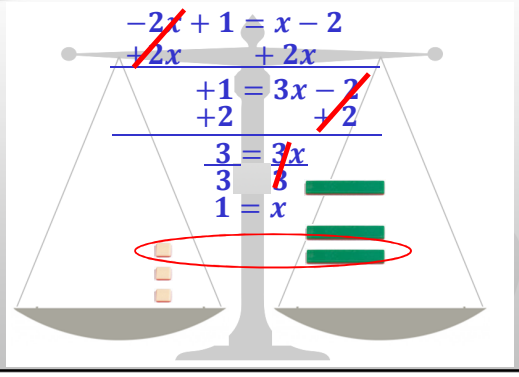

ipdae
BY EDUCATORS FOR EDUCATORS
Solving Equations
$-2x + 1 = x - 2$
$+2x \qquad + 2x$
$+1 = 3x - 2$
$+2 \qquad + 2$


ipdae
BY EDUCATORS FOR EDUCATORS
Solving Equations
$-2x + 1 = x - 2$
$+2x \qquad + 2x$
$+1 = 3x - 2$
$+2 \qquad + 2$
$3 = 3x$

ipdae BY EDUCATORS FOR EDUCATORS

Solving Equations

$$-2\cancel{x} + 1 = x - 2$$
$$\cancel{+2x} \qquad + 2x$$
$$+1 = 3x - \cancel{2}$$
$$+2 \qquad \cancel{+2}$$
$$\frac{3}{3} = \frac{\cancel{3}x}{\cancel{3}}$$
$$1 = x$$



ipdae BY EDUCATORS FOR EDUCATORS

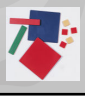
Solving Equations

## Your Turn

Now, it's your turn. Within your groups, form triads or pairs and start solving the equations using algebra tiles. Designate one person who will solve the equation on the writing board while others model the solution.

1. Rene is to divide seven apples equally to three of his children and keep one for himself. How many apples will each child receive?

2. The sum of three consecutive positive integers is 6. Find the value of the smallest integer.



ipdae BY EDUCATORS FOR EDUCATORS

Solving Equations

## Additional Practice

Now, it's your turn. Within your groups, form triads or pairs and start solving the equations using algebra tiles. Designate one person who will solve the equation on the writing board while others model the solution.

A. $3x - 1 = 8$ $\quad x = 3$

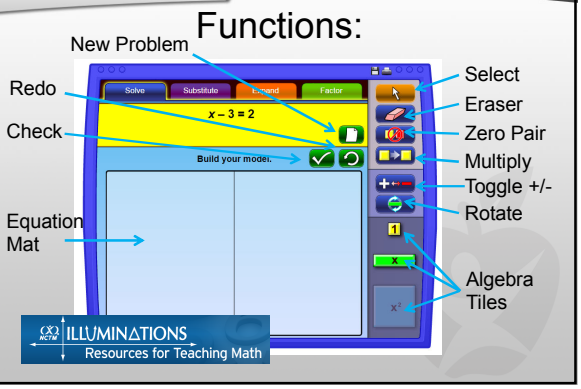
B. $x + 1 = 3x + 5$ $\quad x = -2$

C. $-4x - 3 = -7$ $\quad x = 1$

$x = 3$
$x = -2$
$x = 1$

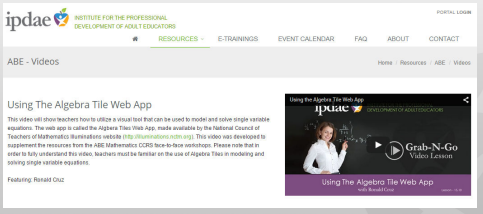
## The Algebra Tiles Web App

## Functions:






## IPDAE Grab and Go



ipdae BY EDUCATORS FOR EDUCATORS

## Check out the IPDAE Website.

ipdae BY EDUCATORS FOR EDUCATORS

## Summary

# Wrap-Up

In this workshop:

- We discussed and apply the major shifts in CCR Standards through various strategies and activities.
- We unpacked the standards and participated on lesson activities that will help in standards implementation.
- We identified integrated standards of mathematical practices in lesson activities.
- We incorporated the use of manipulatives in presenting mathematics content and practicing mathematical skills within 5 domains of CCRS.
- We used appropriate vocabulary and representations of mathematics concepts.

ipdae BY EDUCATORS FOR EDUCATORS

## Door Prize!

Best Hand in Poker

Training Evaluation