Module: Mathematics

Lesson Title: What's My Area? Geometry and Problem Solving

Standards for the Lesson

Florida Adult Basic Education Mathematics Standards
Understand concepts of area measurement and relate area to multiplication and addition. (CCR.MA.ABE.3.2.5) Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. (CCR.MA.ABE.3.2.6) Solve mathematical and real-world problems involving area, surface area, and volume. (CCR.MA.ABE.3.3.4) Solve mathematical and real-world problems involving angle, measure, area, surface area, and volume. (CCR.MA.ABE.4.4.2)

Interpreting the Standard

1 Standards	2 Skills Included in the Standard	3 Concepts Included in the Standard	4 Through a Particular Context	5 Cognitive Demand/Levels of Thinking	6 Sample Activity
Understand concepts of area measurement and relate area to multiplication and addition. (CCR.MA.ABE.3.2.5)	understand relate	concepts of area area to multiplication and addition	Use of manipulatives to explore and discover and the use of real-world problems	DOK 2 DOK 1	Have students determine area by counting square units. Have students work with geoboards in order to explore how perimeter and area relate Have students solve real-world problems by determining perimeter and area of common shapes.
Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures (CCR.MA.ABE.3.2.6)	recognize distinguish	perimeter perimeter vs. area		DOK 1 DOK 1	
Solve mathematical and real-world problems involving area, surface area, and volume. (CCR.MA.ABE.3.3.4)	solve	area, surface area, volume problems		DOK 2	

Solve mathematical and real-world problems involving angle, measure, area, surface area, and volume (CCR.MA.ABE.4.4.2)	solve	angle, measure, area, surface area, volume problems		DOK 2	

Objectives of the Lesson

Students will:

- Use geoboards to pictorially display geometric concepts
- Define a polygon
- Draft solutions to problems regarding area and perimeter using geoboards or dot paper
- Understand the concept of area, including that area is:
    - Two dimensional
    - Measured in square units
    - Found through multiplying the base times the height of a regular polygon

Materials

- Geoboards and rubber bands for each student
- Dot paper
- Chart paper/markers or white board
- **Handout A: Creating Building Plans: Perimeter and Area**
- **Handout B: Geoboards: Let's Get Started**
- **Handout C: Creating Building Plan: Perimeter and Area**
- **Handout D: Area and Perimeter: Is There a Connection?**
- **Handout E: Centimeter Dot Paper**

Instructional Plan

Overview

The development of geometric reasoning and spatial sense is necessary in order for students to understand the geometric aspects of their surroundings and the concepts and relationships of shapes. Because measurement and geometry are used prevalently in real life, it is important to take advantage of these everyday situations to help students build confidence and competency in these areas.

This lesson is designed to assist students in exploring the "whys" of geometric formulas by using manipulatives/hands-on practice through the use of geoboards. The standards for solving mathematical problems involving perimeter and area are part of the standards for ABE 2, 3, and 4. This lesson will provide problems at various complexity levels dependent on the academic level of the student.

Process

Introduce the lesson by asking students the following questions:

- Have any of you ever had to install a tile floor or seen one installed? (Students should share how tiles are installed – generally in a square format)
- What happens if the number of tiles needed to tile the floor do not fit perfectly? (Students should answer that they would have to be cut or shaped to fit the area.)
- What formulas do you know that deal with area and perimeter? Are they the same formulas for different shapes? (Write the students' answers on the board, including correct formulas provided.)
- Why do the formulas work? (Students may not be able to answer this question. Share with students that they will learn more about the "why" in this lesson.)

Share with students that today they are going to use geoboards to experiment with different types of polygons. Discuss that geoboards are a way for them to better understand how the formulas of area and perimeter work.

Provide each student with a geoboard. Geoboards are an inexpensive manipulative that provide students with a concrete way of learning about geometric concepts. Virtual geoboards are also available, such as the geoboards located on the National Library for Virtual Manipulatives at: http://nlvm.usu.edu/en/nav/vlibrary.html

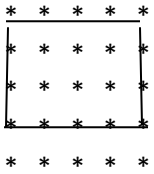
If geoboards are not available, you can provide students with a paper version of a geoboard using the template provided in **Handout A: Geoboard Template** or the dot paper located in **Handout E: Centimeter Dot Paper**.

Please note that for maximal understanding, actual geoboards or virtual geoboards are preferable and are inexpensive to purchase.

Introduce the use of geoboards by having students locate as many squares as they can on their geoboard. To get students started, show them a square that you have located by placing the rubber band on one pin and connecting it to other pins to form the perimeter of a square. Remember, it is visually challenging for students to find all of them, especially the ones whose sides are not horizontal or vertical or those which reside "within" other squares. Once they have "located" the various squares, ask students how drawings could be helpful in assisting them to find area and perimeter of both traditional and non-traditional shapes. Share with students that often we see irregular shapes in the real world, such as the shape of rooms or yards. Ask students if they can identify other times when they have had to determine the perimeter or area of an irregular shape.

Next, provide students with **Handout B: Geoboards: Let's Get Started**! Have students create each of the figures on their geoboards. Show students how to determine perimeter for each of the shapes by counting the number of units from one pin to the next. Ask students why the perimeter for their figures may be different from those of their classmates. Students should answer that the size of their figure impacts the perimeter.

Model for students how they can determine the area of different types of polygons by counting the number of square units within the shape. Show students how to accomplish this task by setting up a rectangle that has a width of 4 and a height of 3, such as the following drawing:

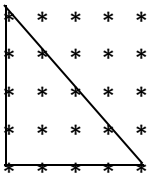


Model for students how you count the number of square units within the shape to get a total of twelve square units. Have students count how many points "surround" the rectangle (14). Discuss that this is the perimeter of the rectangle. Now have students count how many "square units" are within the rectangle (12). Discuss that this is the area of the rectangle.

Provide students with additional problems where they need to create a square or rectangle and figure the perimeter and/or area. Write the following on the board:

Side + Side + Side + Side = Perimeter
Length x Width = Area

Next, model for students how you can also determine the area of a different type of polygon, such as a right triangle by counting the number of squares within the shape. Make sure to show students how one-half of a square unit and one-half of a square unit equate to a whole square unit.



The answer for the above shape is 8 square units.

Have students set up depictions of traditional polygons, such as a rectangle, parallelogram, triangle, and trapezoid. You may wish to provide students with different measurements, such as set up a rectangle that has a width of 5 and a height of 4.

After students have practiced working with both perimeter and area, ask the following question: *What is the largest possible area of a rectangle if its perimeter is 20?*

Answer: The largest area of any rectangle with a given perimeter will occur when a square is formed. With a perimeter of 20, a square will form an area of 25. This is the largest area a rectangle can have with a perimeter of 20.

To close the lesson, share with students how the skills of area and perimeter have a real-world connection. Provide students with ***Handout C: Creating Building Plans: Perimeter and Area***. Have the students explore the many possible shapes of a polygon and how to figure the perimeter and area. Have students share the shape of their shed for Fred with the class.

Sample Debriefing Questions

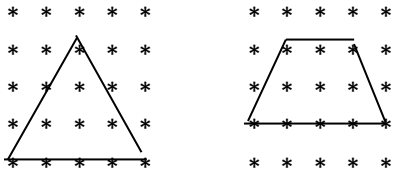
- What is the difference between perimeter and area? How would an architect or construction worker use these different measurements? (Perimeter is the measure of the distance around an object. Area is the measure of all the space inside an object.)
- How do you find the area? How do you find the perimeter? Compare your method with your classmates.
- Are perimeter and area connected? How? (Student answers may vary.)

Modifications for Different Levels

Students who are struggling with the concepts of perimeter and area may not be ready to move from concrete to abstract problems and may need to spend more time exploring with the geoboard. Students may also be more successful in developing the correct answer by using the geoboards or dot paper on which to write their answers. The use of manipulatives can be used at any level of learning. Students need to be able to see the concrete before they can achieve skills at the representative and abstract levels.

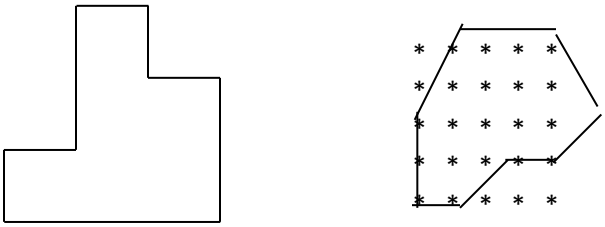
For students who are beginning to learn about perimeter and area, provide them with simple polygon shapes, such as squares and rectangles.

For higher-levels of proficiency, add shapes such as triangles and parallelograms.



After spending time calculating the area of squares, rectangles, and triangles, provide students with a more complicated polygon or irregular two-dimensional shape that they need to partition.

Examples may be something similar to the following:



Have students at higher levels compare and contrast area and perimeter by completing **Handout D: Area and Perimeter: Is There a Connection?** To debrief the activity, have students share their answers. Have them summarize what they have learned and share it with the class.

Resources for the Classroom

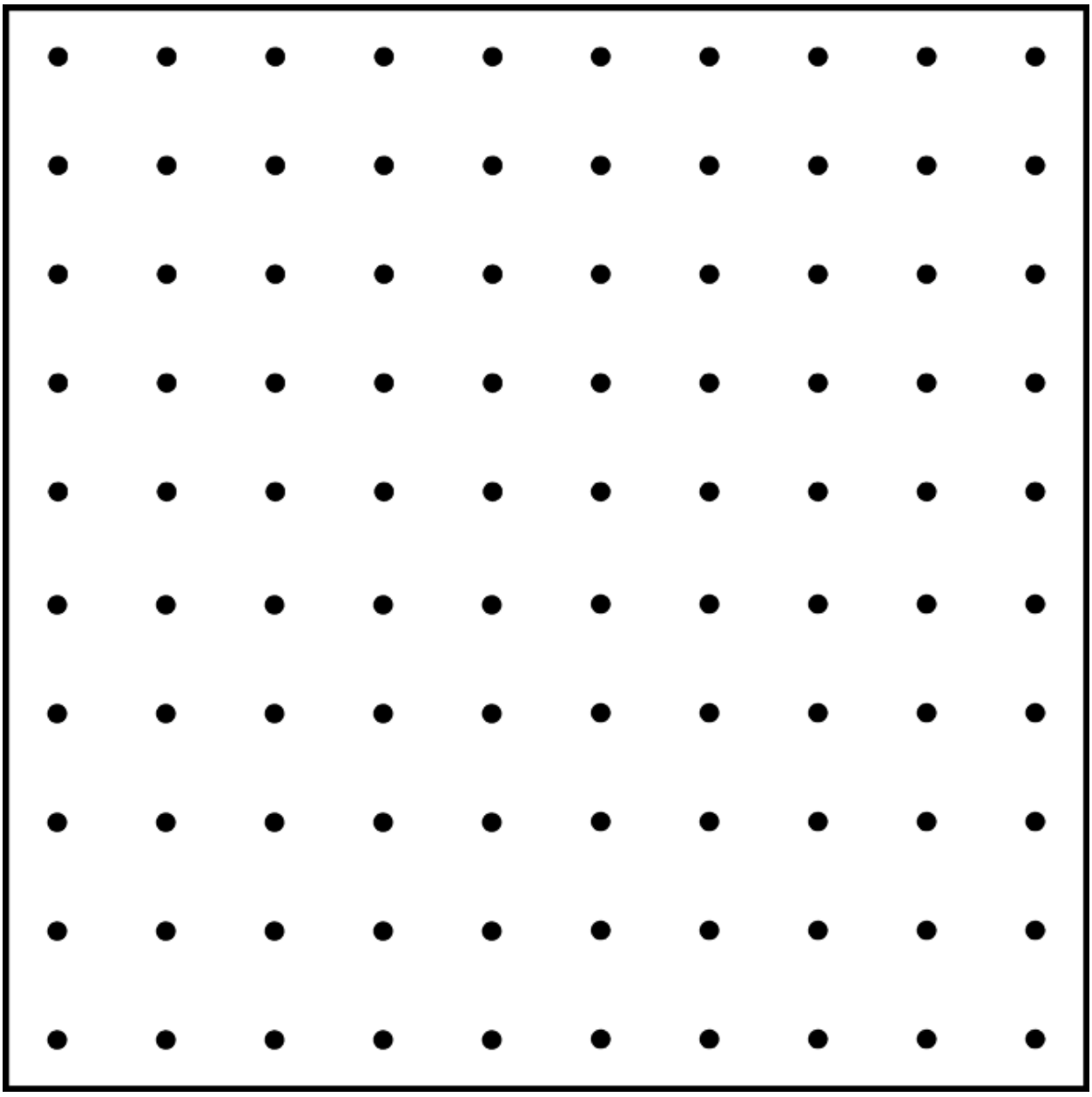
There are great resources for using geoboards in the classroom. To get started, you may wish to access the following sites before teaching this lesson:

Exploring Area with a Geoboard. Learning Math. Annenberg Learner.
http://learner.org/courses/learningmath/measurement/session6/part_b/index.html

Virtual Manipulative Geoboard. Utah State University. National Library for Virtual Manipulatives.
http://nlvm.usu.edu/en/nav/vlibrary.html

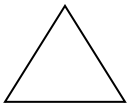
Scavo, Tom. Geoboards in the Classroom. The Math Forum.
http://mathforum.org/trscavo/geoboards/contents.html

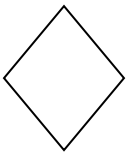
Handout A: Geoboard Template

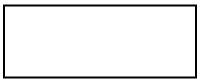


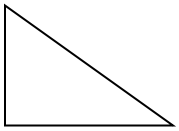
Handout B: Geoboards: Let's Get Started

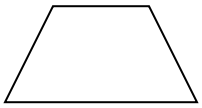
To get you started, make an interesting figure on your geoboard. How many of these figures can you show on your geoboard?

Isosceles triangle 

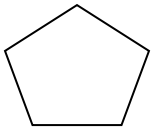
Diamond 

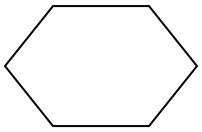
Rectangle 

Right triangle 

Trapezoid 

Parallelogram 

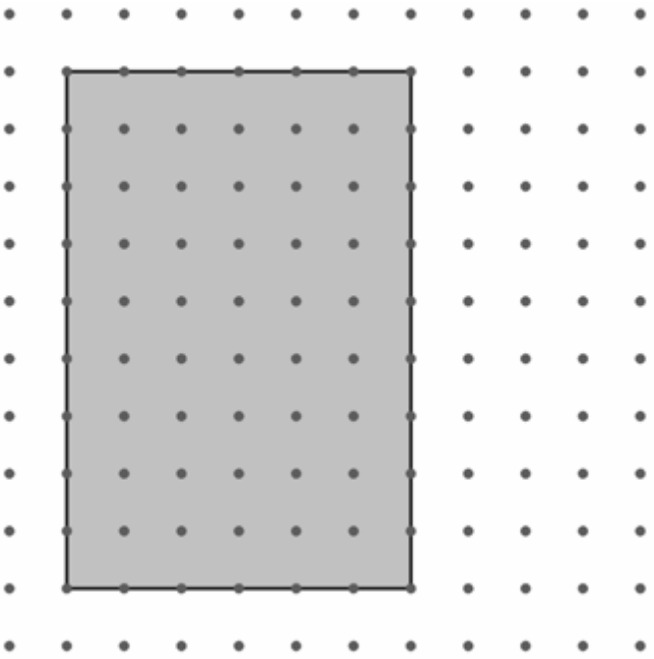
Pentagon 

Hexagon 

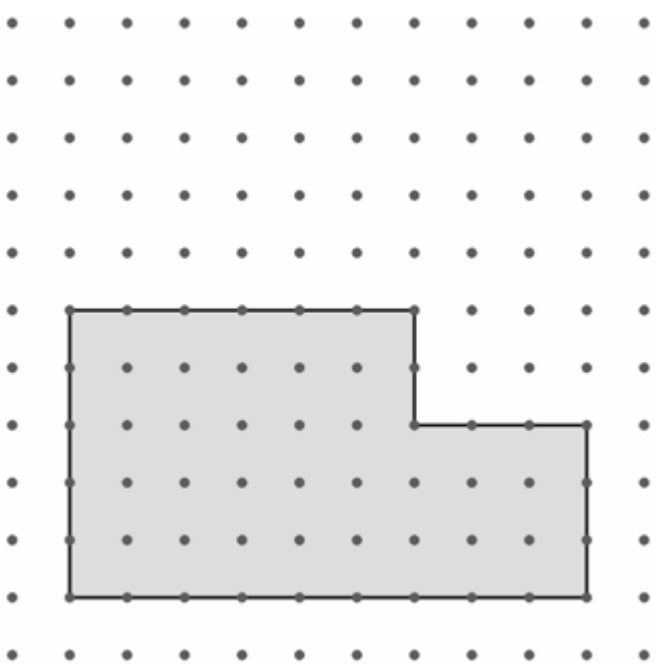
Handout C: Creating Building Plan: Perimeter and Area

Samuel is building outdoor sheds for his customers. He wants to offer different shapes and sizes to his customers so that they have a selection from which to choose. He has drawn four different figures, now all he has to do is figure the perimeter and area of each shed so that he can figure his building cost. Help Samuel calculate the measurements.

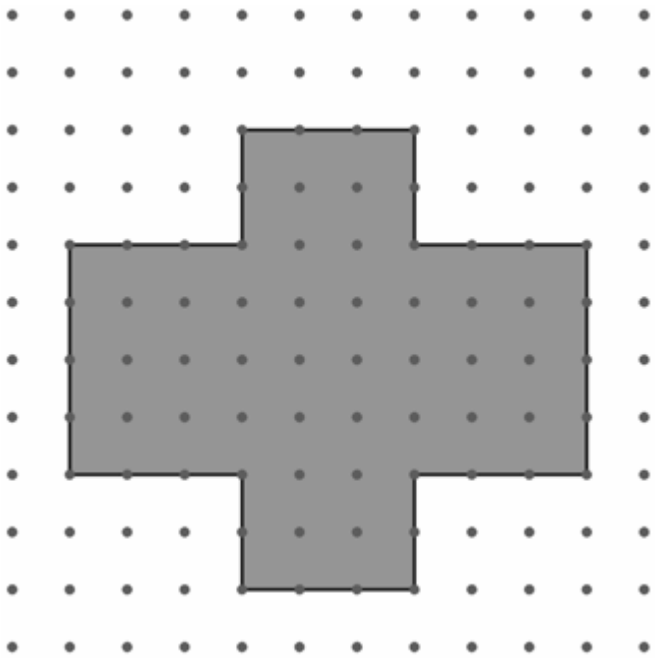
Perimeter: _____ Area: _____



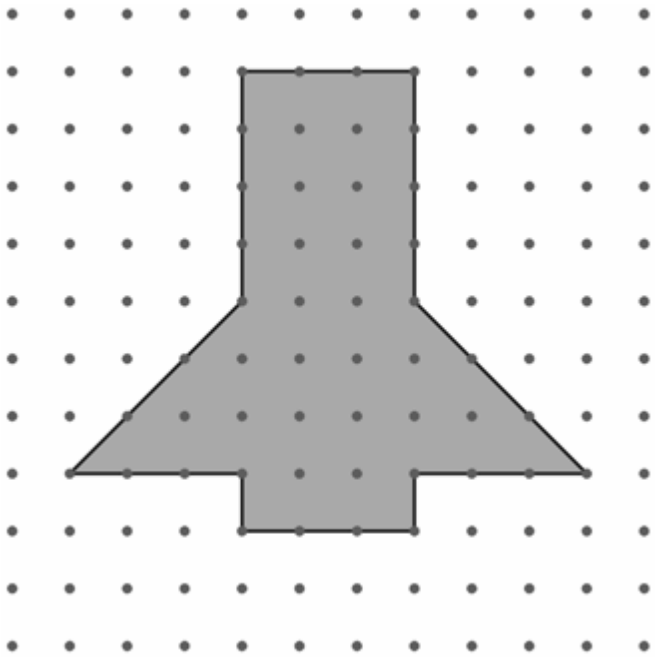
Perimeter: _____ Area: _____



Perimeter: _____ Area: _____

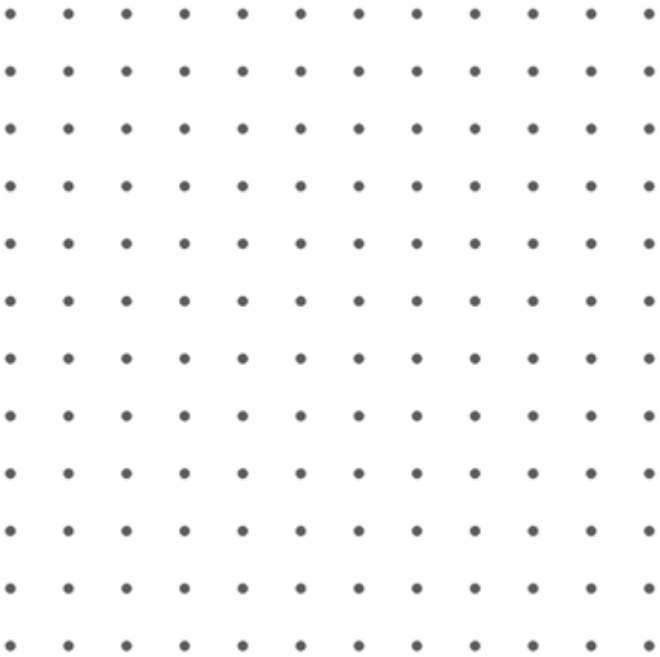


Perimeter: _____ Area: _____

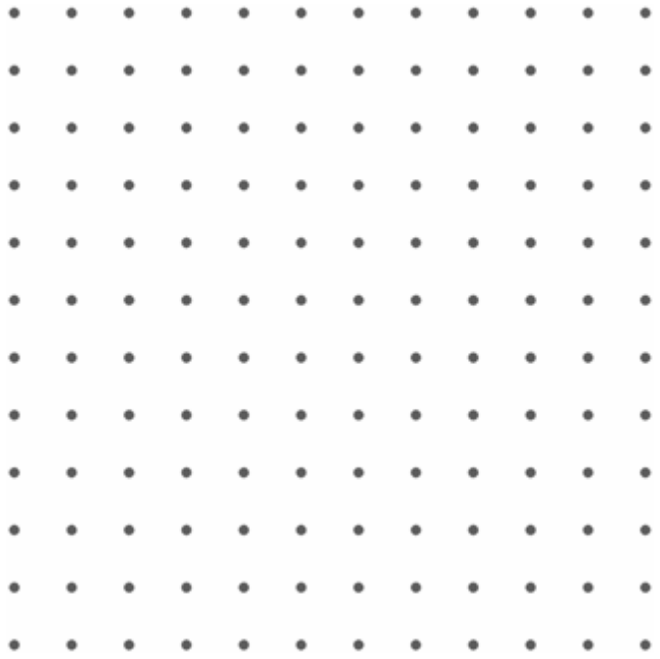


Can you create other shapes for a shed for Samuel? Draw your shapes on the paper below and provide Samuel with the perimeter and area in square units for each.

Perimeter: _____ Area: _____



Perimeter: _____ Area: _____



Handout D: Area and Perimeter: Is There a Connection?

What is the relationship between area and perimeter? In this activity we will investigate this relationship.

On your geoboard use a small rubber band to create a square using four adjacent pins. This square has the **area** of 1 square unit because 1 square is enclosed by the rubber band. You can cover the region enclosed by the rubber band with 1 square. The **perimeter** of this square is the sum of the four line segments surrounding the square. The perimeter is 4 units. (Note that area is always expressed in square units such as square inches or square meters while perimeter is always expressed in linear units such as inches or meters.) What will happen to the perimeter and area of the square if you double the length of a side? Use a larger rubber band to create a square that has a length of 2 units on a side. Record the perimeter and area in the table below.

Repeat this for a square that is 3 units on a side and a square that is 4 units on a side.

Length of Side	Area	Perimeter
1 unit	1 square unit	4 units
2 units		
3 units		
4 units		

1. If you increase the perimeter of a square, does the area always increase?

2. Use the patterns in the table to predict the area and perimeter of a square that is 5 units on a side. Explain how you arrived at your answers.

3. What is the only square with the property that the area and the perimeter are numerically equal? How do you know this is the only such square?

4. The 2 by 2 square has a perimeter of 8 units. Are there any other rectangles (remember – a square is a rectangle) with the same perimeter? What is the area of each such rectangle? Use the geoboard to assist you.

5. The 3 by 3 square has a perimeter of 12 units. Are there any other rectangles with the same perimeter? What is the area of each such rectangle?

6. The 4 by 4 square has a perimeter of 16 units. Are there any other rectangles with the same perimeter? What is the area of each such rectangle?

7. Based on the results of your experiment is it possible for two rectangles to have the same perimeter but different areas? Justify your answer.

8. Based on the results above is it possible to increase the perimeter of a rectangle without increasing the area? Justify your answer.

Handout D: Area and Perimeter: Is There a Connection? – Sample Answers

1. *If you increase the perimeter of a square, does the area always increase?*
   Remember, a rectangle is a special type of square, so students may share the following type of problem:
   - A square with all sides being 3 units would have a perimeter of 12 and an area of 9
   - A rectangle (a type of square) has sides of 1 and 6, so the perimeter would be 14 and the area would be 6.

2. *Use the patterns in the table to predict the area and perimeter of a square that is 5 units on a side. Explain how you arrived at your answers.*
   Area = 25 square units; perimeter = 20
   Students' answers will vary regarding their explanation.

3. *What is the only square with the property that the area and the perimeter are numerically equal? How do you know this is the only such square?*

   The answer is 4. Students may make a table, such as the following, to figure out this problem.

Side length of square	Area	Perimeter
1	1	4
2	4	8
3	9	12
4	16	16
5	25	20
6	36	24

4. *The 2 by 2 square has a perimeter of 8 units. Are there any other rectangles (remember – a square is a rectangle) with the same perimeter? What is the area of each such rectangle? Use the geoboard to assist you.*

   Answers may include:
   Sides 1 and 7 for an area of 7 square units
   Sides 3 and 1 for an area of 3 square units

   Students may include perimeters that have fractional parts of a unit, such as: sides .5 and 1.5 for an area of .75 square units

5. *The 3 by 3 square has a perimeter of 12 units. Are there any other rectangles with the same perimeter? What is the area of each such rectangle?*

   Sample answers may include:
   Sides 1 and 11 for an area of 11 square units
   Sides 2 and 4 for an area of 8 square units

   Students may include perimeters that have fractional parts of a unit, such as: sides 1.5 and 4.5 for an area of 6.75 square units.

6. *The 4 by 4 square has a perimeter of 16 units. Are there any other rectangles with the same perimeter? What is the area of each such rectangle?*

   Sample answers may include:
   Sides 1 and 15 for an area of 15 square units
   Sides 2 and 8 for an area of 16 square units

   Students may include perimeters that have fractional parts of a unit, such as: sides 4.5 and 3.5 for an area of 15.75 square units.

7. *Based on the results of your experiment is it possible for two rectangles to have the same perimeter but different areas? Justify your answer.*

   Yes. Students should provide examples from their tables/calculations.

8. *Based on the results above is it possible to increase the perimeter of a rectangle without increasing the area? Justify your answer.*

   Yes. Students should provide examples from their tables/calculations.

Handout E: Dot Paper

Centimeter Dot Paper