MATH MANIPULATIVES

Preparing for the College and Career Readiness Standards for Adult Education

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Table of Contents

PDAE
From the field, for the field.

Math Manipulatives

Teachers are always interested in looking for ways to improve their teaching and to help students understand mathematics. Research in England, Japan, China, and the United States supports the idea that mathematics instruction and student mathematics understanding will be more effective if manipulative materials are used.

Mathematics manipulative is defined as any material or object from the real world that students move around to show a mathematics concept.

Research indicates that students of all ages can benefit by first being introduced to mathematical concepts through physical exploration. By planning lessons that proceed from concrete to pictorial to abstract representations of concepts, you can make content mastery more accessible to students of all ages.

Long-Lasting Understandings

With concrete exploration (through touching, seeing, and doing), students can gain deeper and longer-lasting understandings of math concepts. For example, students can explore the concept of least common multiple with integer bars. They can place the integer bars side by side, experiment, and discover how to create a combination of bars that are the same length. Once students have a concrete understanding of the concept of greatest common factor (GCF) as matching lengths, they will find it easier to use a number line or make lists to identify the GCF. Similarly, if students use grid paper, pencils, and scissors to discover the formulas for computing the areas of parallelograms, triangles, or trapezoids, the formulas will make sense to them and they will be more likely to remember the formulas.

Using manipulative materials in teaching mathematics will help students learn:

1. To relate real-world situations to mathematics symbolism
2. To work together cooperatively in solving problems
3. To discuss mathematical ideas and concepts
4. To verbalize their mathematics thinking
5. To make presentations in front of a large group
6. That there are many different ways to solve problems
7. That mathematics problems can be symbolized in many different ways
8. That they can solve mathematics problems without just following teachers' directions.

Managing Manipulatives

Using manipulatives can present classroom management challenges. Teachers find that manipulatives can get lost or broken. Students sometimes use manipulatives for other than the intended purpose. Distributing the manipulatives can take time, but the following guidelines can assist the teacher in using them more effectively.

Set Up Simple Storage Systems - Set up a simple system to store the manipulatives. Some teachers arrange shelves or cupboards with plastic boxes or snap-and-seal bags. Others place their materials in the center of tables or desks. Clearly label your storage containers. Make sure students understand the system and have easy access to it.

Establish Clear Rules - Prior to your first use of manipulatives, discuss a clear set of rules for using the manipulatives with your students. You may want to explain what manipulatives will be used for and include the following information:

- appropriate uses for learning
- handling
- storage
- distribution and return
- student roles and responsibilities

Structured Learning Experiences

The key to successful hands-on activities is to provide a structured learning experience in which students learn how to use manipulatives. To maximize learning, always provide three levels of practice.

- Modeled Tasks - Before distributing materials, provide clear instructions and model the tasks the students will carry out. If it is appropriate, you can invite students to help you model. For example, if the students are going to use fraction bars to complete addition problems, you might have students model using overhead bars, how they completed the process.
- Guided Practice - Give students opportunities to practice prior to working individually or in small groups. If this is the first time the student is handling the manipulative, consider allowing extra time for exploration. You might ask the students to construct the largest possible right angle on a geoboard and give them time to figure out how to work with the pegs and rubber bands. Monitor student practice during this phase to give them the support they need to be successful when they work independently.
- Independent Work - Once students know how to use manipulatives, they can complete problems on their own or in small groups with less support. This is an excellent time to informally assess learning and provide intervention as needed.

By following these guidelines and with time and practice, you will be able to introduce concepts through hands-on activities as easily you can lead pencil-and-paper activities.

Why Don't All Teachers use Manipulatives?

If the findings are so compelling then why don't all mathematics teachers use manipulatives? There are several possible reasons why more mathematics teachers do not use manipulatives in their lessons, namely:

1. Lack of training, many teachers feel that they do not know how to teach using manipulatives and, therefore, are not comfortable using manipulatives and hesitate to use them in the classroom.
2. Availability of funds to purchase manipulatives or time to develop the hands-on materials.
3. Lessons using manipulatives may perhaps be noisier and not as neat. Using manipulatives works nicely in a cooperative learning setting. It is a good idea to use plastic cups or ziplock sandwich bags as a way to keep manipulatives organized.
4. A fear of the breakdown in classroom management. Manipulatives require a great deal of prior planning and organization.

Materials for Creating Manipulatives

There are many different types of manipulatives available for purchase. However, you can access free and inexpensive materials for making your own classroom manipulatives. The following are some examples of free or inexpensive materials that can be accessed.

Free Items

- cardboard
- styrofoam food trays from the grocery store
- plastic serving dishes (If you line them with felt, they make quiet surfaces for rolling dice.)
- tissue boxes (for making 3D geometric shapes)
- refrigerator magnets from businesses (cut them up and glue them to manipulatives.)
- small or large boxes (reach inside and trace the inside edges with a finger to get a 3D feel.)
- books of outdated wallpaper samples
- lost and spare buttons from clothing and notions stores (or students' homes)
- road maps (often available from AAA at the end of the year)

Inexpensive Items

- paper (regular, cardstock, or poster board)
- small food items: dried beans (to be marked), cereal, marshmallows, uncooked pasta (Try tossing the pasta in a plastic bag with vinegar and food coloring to make manipulatives of different colors. You can write numbers on the pasta, too.)
- rubber liner
- plastic needlepoint material
- toothpicks
- popsicle sticks
- magnetic tape
- ziploc bags
- dice
- cards
- 3x5 cards
- tape measurers
- rulers
- protractors

Types of Math Manipulatives to Create

- Power of Ten blocks
- Counters
- Cuisinaire rods
- Clocks
- Play money
- 3D cubes or Algeblocks
- Pattern blocks
- Dice
- Number tiles
- Number lines
- Measurement Tools
- Compass
- Protractors
- Rulers
- Tangrams
- Geoboards

Benefits to Students in Creating Their Own Manipulatives

Although you, the teacher, can create classroom sets of manipulatives, there are many reasons for providing students with the opportunity to “create” their own manipulatives, such as:

Creating manipulatives is empowering: It provides students with a feeling of:

- ownership;
- independence; and
- resourcefulness.

It can be a creative process.

- Students apply math skills when planning, measuring, and making efficient use of resources. The more they practice measuring precisely for meaningful purposes, the better their skills are in this area.
- The process can build real-world connections with a sense that math is everywhere and that everyday items have hidden potential as math tools.
- Units of measurement are a convention, but students can measure creatively for their own purposes.
- Students can create their own units of measurement.
- Objects can represent one unit or a group of units (such as 10 or 100).
- Manipulatives can resemble a specific type of item or a specific value; they can also be an abstract representation. (For instance, a dime doesn't *look* like it is worth 10 pennies.)
- Students have the opportunity to engage in cooperative learning and sharing.
- Students of any age can make manipulatives; adult students can begin with simple objects for learning basic operations and gradually move up to more complex tools, such as algebra tiles.
- It provides students with a chance to explore and discover mathematical concepts in an open-ended way before using them for specific math activities.

Manipulative Templates

Base 10 Blocks

Objective: Color and cut out your own Base Ten blocks and use them to illustrate addition, subtraction, multiplication, and division problems.

Materials:

- Base Ten template
- colored posterboard
- scissors
- magnetic tape (optional)
- Base Ten mat (optional)

How to Create:

- Print the Base Ten template onto posterboard or onto colored paper and then laminate the paper for durability.
- Optional: Cut small pieces of magnetic tape and press a piece onto the back of each one-unit. Press two or three pieces onto the 10-rods, and place one piece in each corner of the 100-squares.

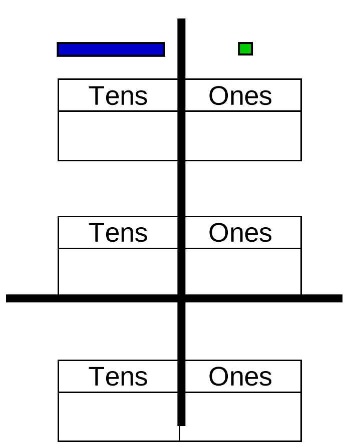
How to Use:

Addition

The basic method for addition consists of pulling the correct type and number of blocks that represent the *addends*, combining them together, and adding the total to obtain the *sum*. The best backdrop to use is the addition/subtraction mat. Each block type should be kept in its corresponding column.

Subtraction

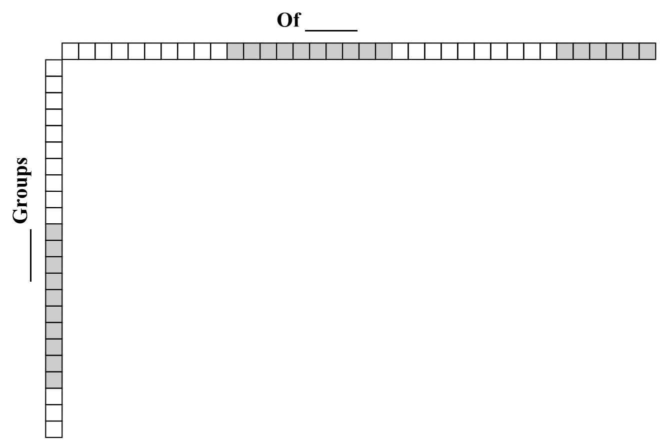
To do subtraction you start by pulling out the correct type and number of blocks that represent the *minuend*. You then proceed to take away and recycle the blocks that represent the *subtrahend*. To do this you may have to break blocks into the next smaller unit. Once the blocks have been recycled you count the remaining blocks to obtain the result or difference. The best backdrop to use for subtraction is addition/subtraction mat.



Multiplication

With multiplication it starts to get more interesting as the students can visualize the area represented by the multiplication of the two factors. For multiplication, you essentially do repeated addition. For example, to multiply 7 x 12, students should indicate on the chart 7 groups of 12. Multiplication using base 10 blocks also provides an excellent way to teach the commutative property - 7 groups of 12 and 12 groups of 7 equal the same answer.

Multiplication can be done using the backdrop shown in the picture below,



When you are done completing the multiplication, you only have to count all the blocks which make up the area of the 7 by 12 rectangle to get the product of the two factors.

Division – Measurement Method

One of the methods to do division using base 10 blocks consists of breaking the dividend into groups the size of the divisor and counting the number of groups. The basic steps, using (84/12) as the example, are:

1. Bring into the working area the blocks that represent the dividend. In this case it would be 8 ten-blocks and 4 one-blocks,
2. Group the blocks into sizes that represent the divisor. In this case the size of each group should be 12 and you should end up with 7 separate groups giving you a quotient or result of 7.
3. You will typically have to “exchange” the larger blocks for “smaller” equivalencies, such as 10 tens for a 100 block.
4. You probably want to start to teach division by choosing numbers that divide exactly with no remainder. However, once the students understand the concept of division, this method (as well as the partition one) are great to introduce the concept of the remainder, i.e., examples when the divisor does not divide the dividend exactly and you end up with a group of blocks that does not fill up the group size requirement.

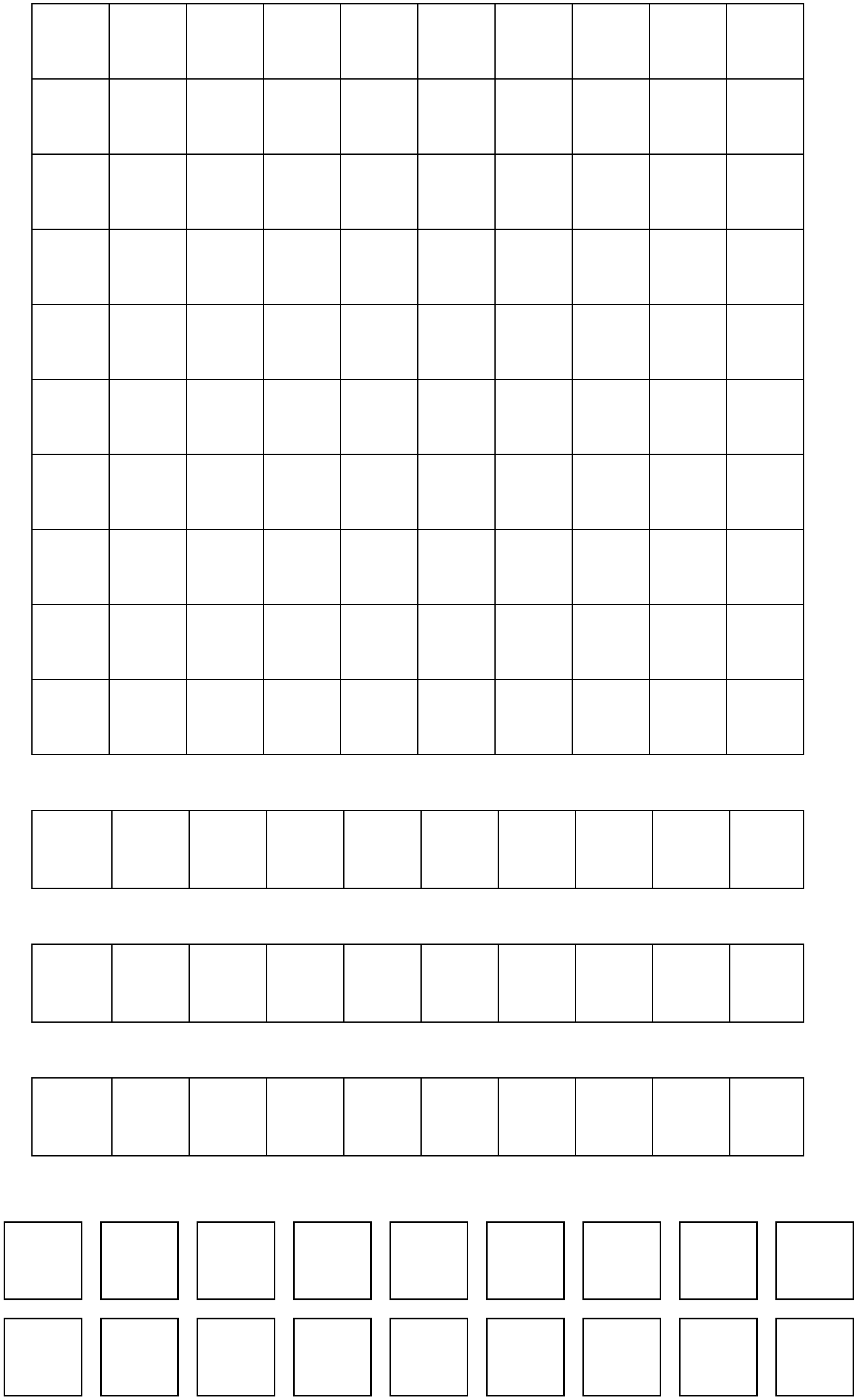
Division – Partition Method

The partition method is a variation of the measurement method. Instead of grouping the blocks into groups of 12 and moving them off, you distribute the blocks, one by one, into 12 different places in the working area. When done moving off all the blocks, you simply count how many blocks ended up in each pile.

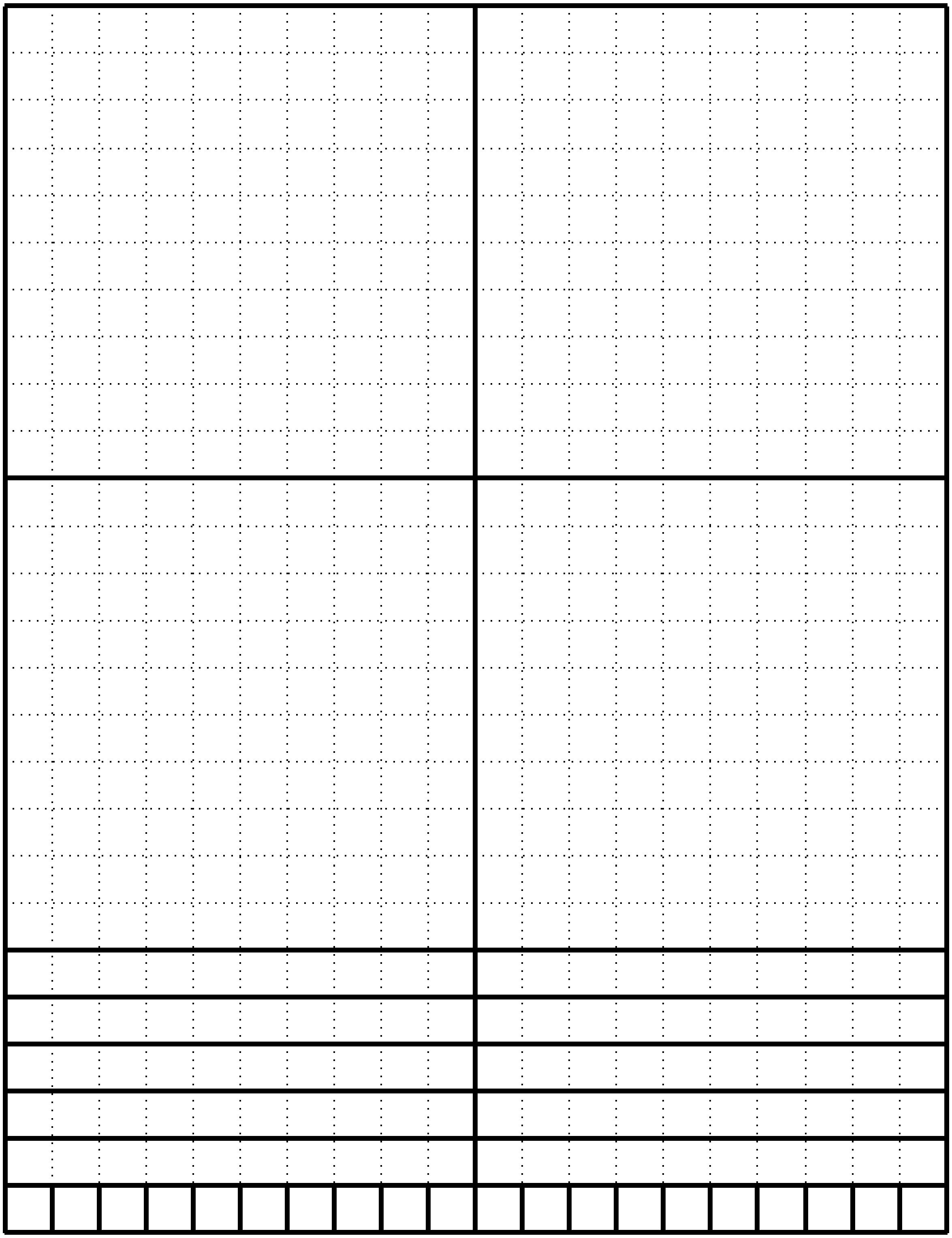
You can distribute more than one block at the same time as long as you divide them fairly. That is, every pile receives the same number of blocks. You can also use this method to illustrate very clearly the concept of remainder by noting the leftovers.

The difference between the two division methods is how the question is asked. Does the student think about how many 12s are in 84 (measurement method) or how many blocks can be placed equally in 12 groups (partition method)?

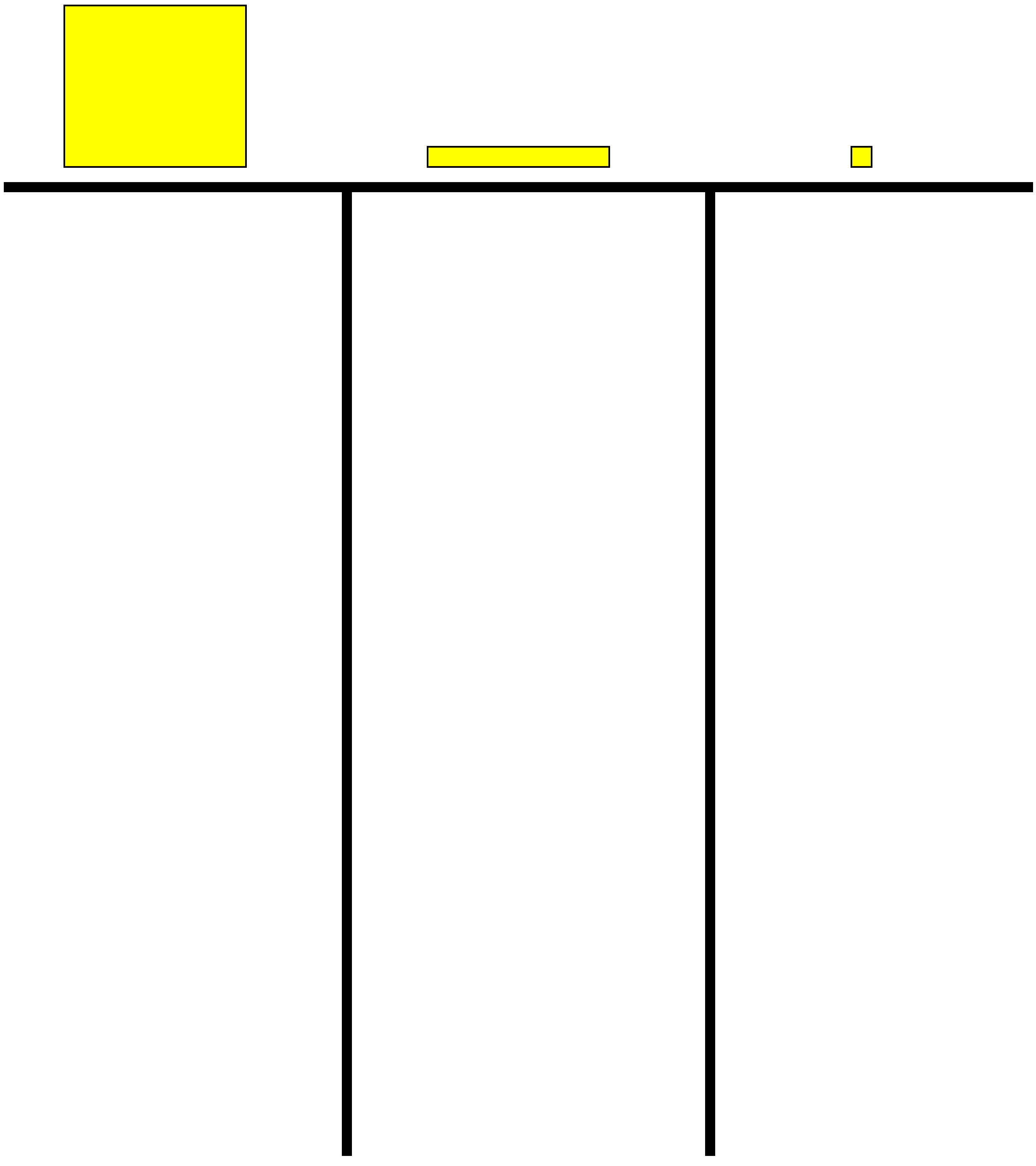
Base 10 Blocks Template



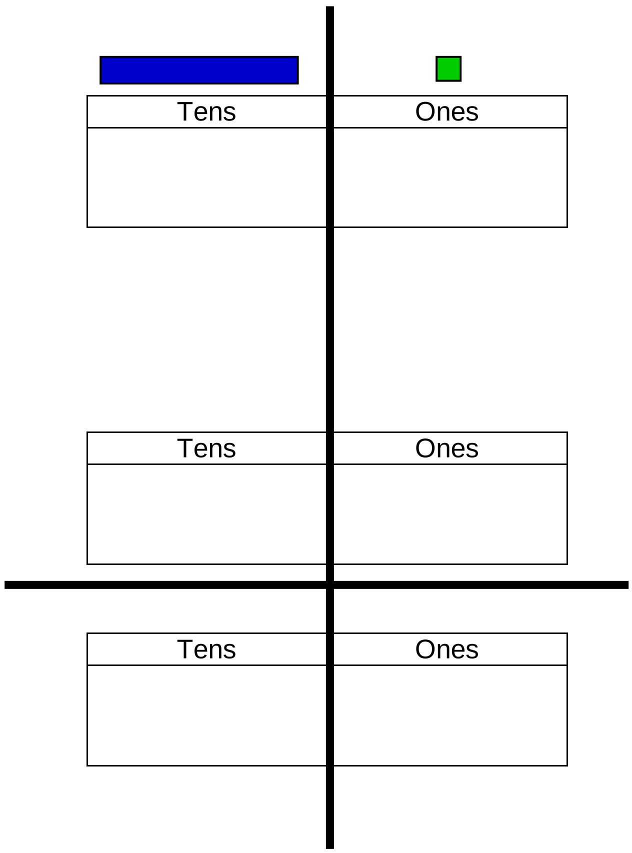
Base 10 Blocks - Cut along the solid lines!



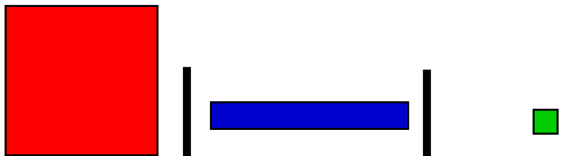
Base Ten Math Template



Base 10 Blocks – Adding and Subtracting with Regrouping Math Template



Base 10 Blocks – Adding and Subtracting with Regrouping Math Template



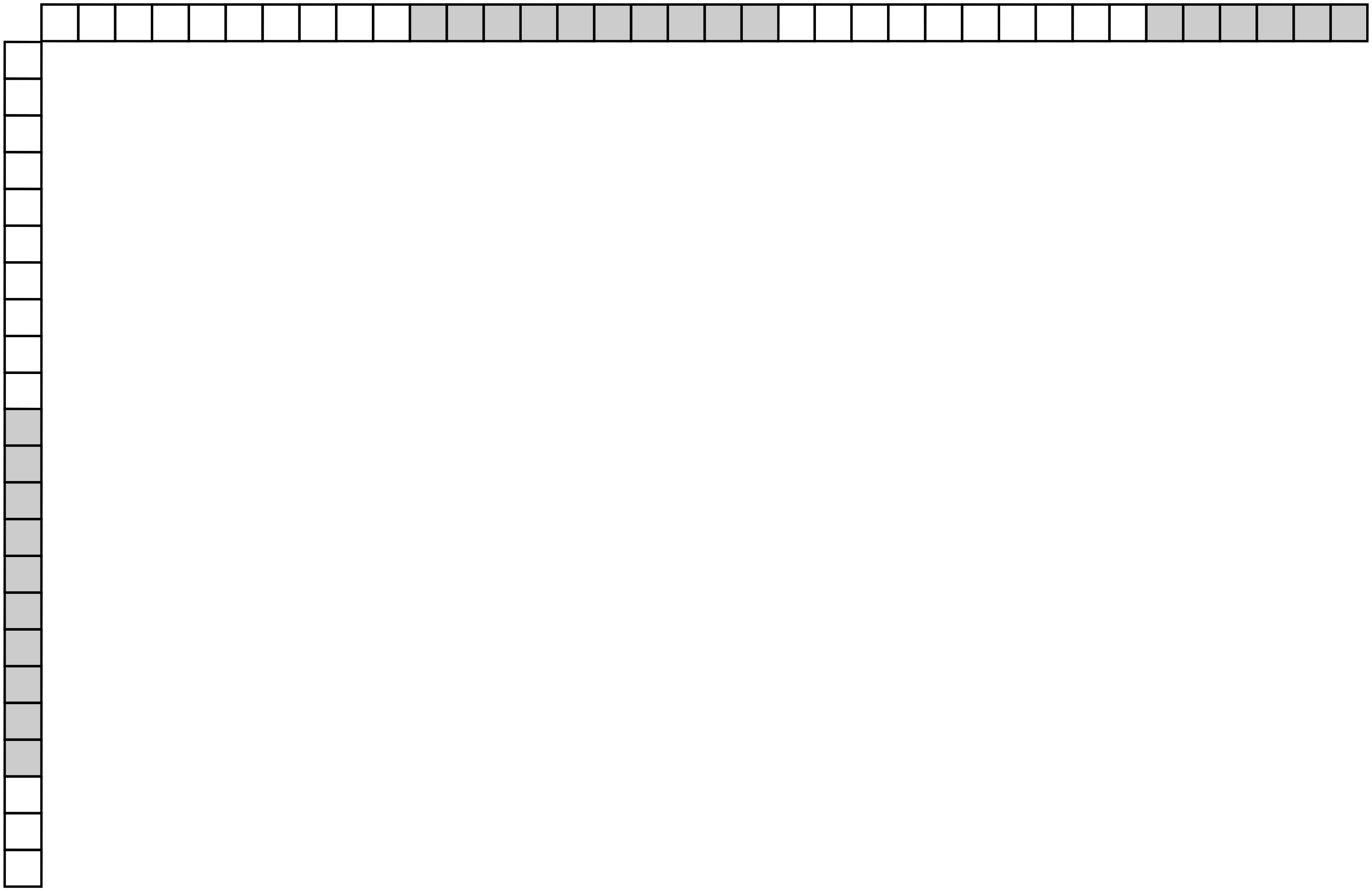
Hundreds	Tens	Ones

Hundreds	Tens	Ones

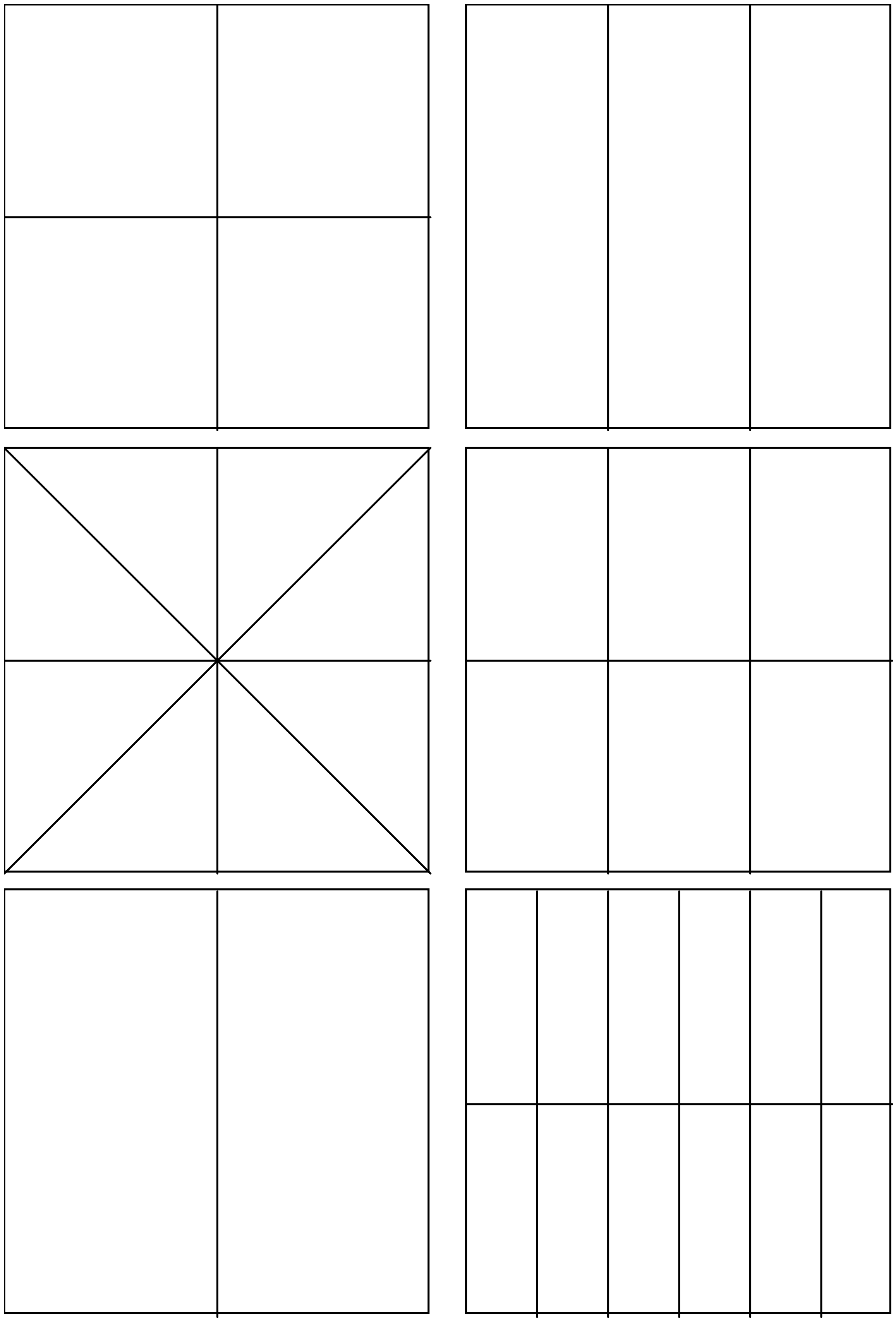
Hundreds	Tens	Ones

Of ______

______ Groups



Fraction Blocks



Fraction Tiles

$\frac{1}{2}$	$\frac{1}{2}$

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

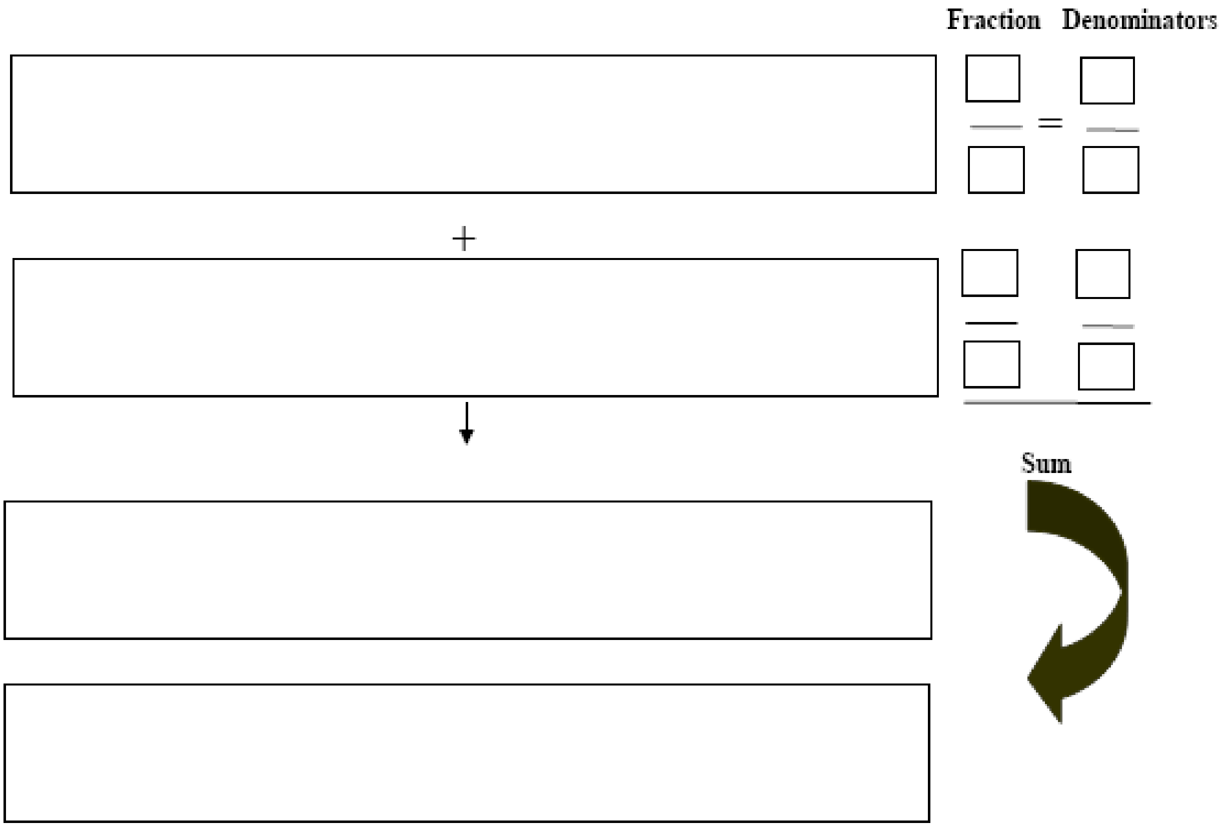
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Fraction Sum Sheet



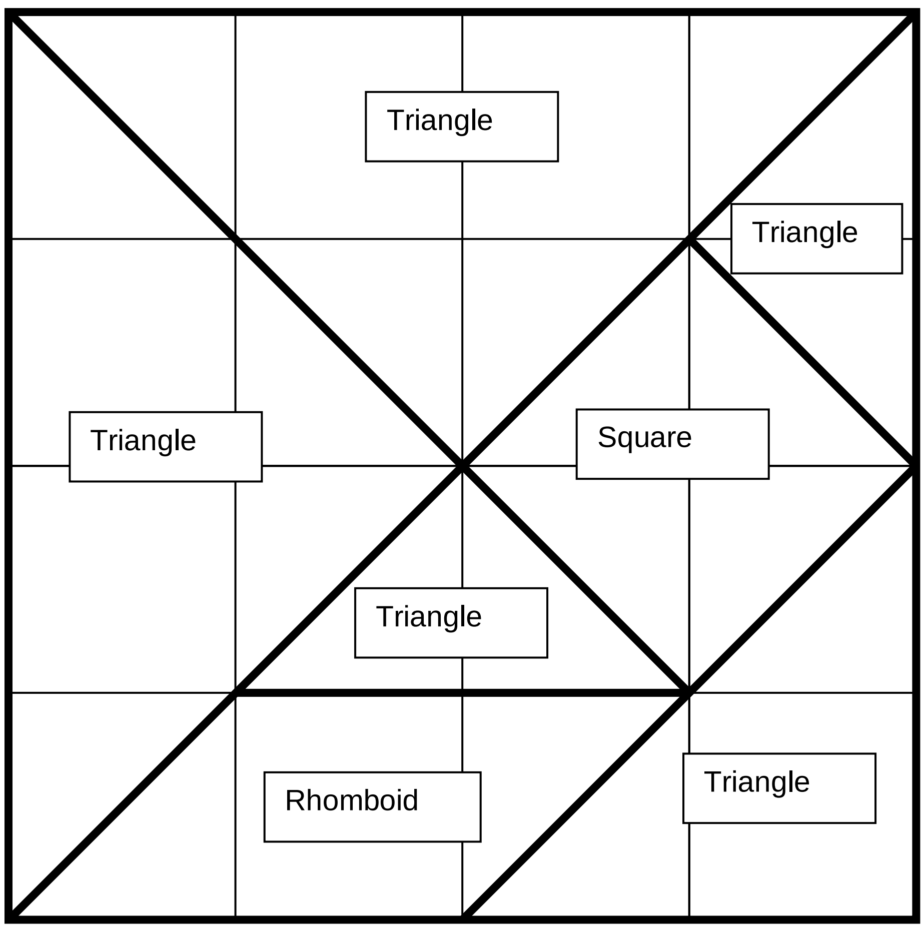
Tangrams

This fun Oriental game can help students learn about geometry. Using seven pieces (five triangles, one square and one rhomboid) the challenge is to arrange them to make up pictures. The rules are: all pieces must be used and pieces must be laid flat with no overlapping.

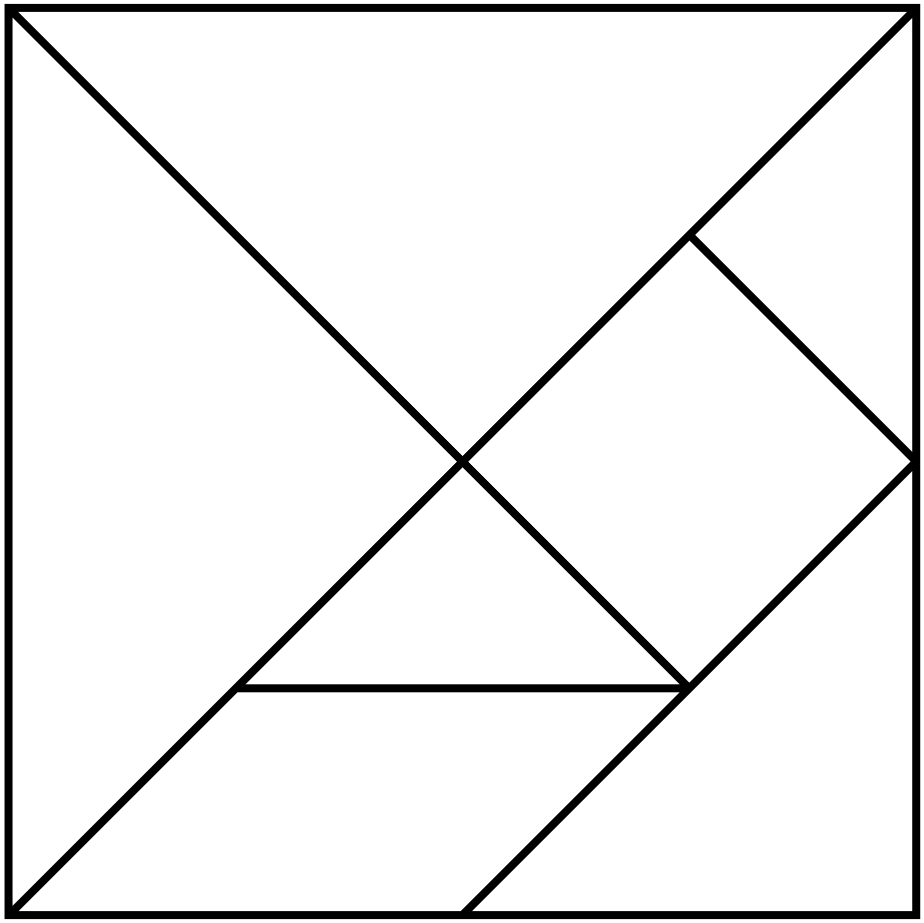
Below is shown how to mark out the cuts on a 4 x 4 grid. Also included are small and large templates that can be used. Print the templates on card stock and cut along the lines.

To create tangram pieces, you will need a four inch (approximately twelve cm.) square, usually about one eighth of an inch (two mm.) thick for a puzzle this size. You can scale this up or down, the four squares by four squares part is the important thing here.

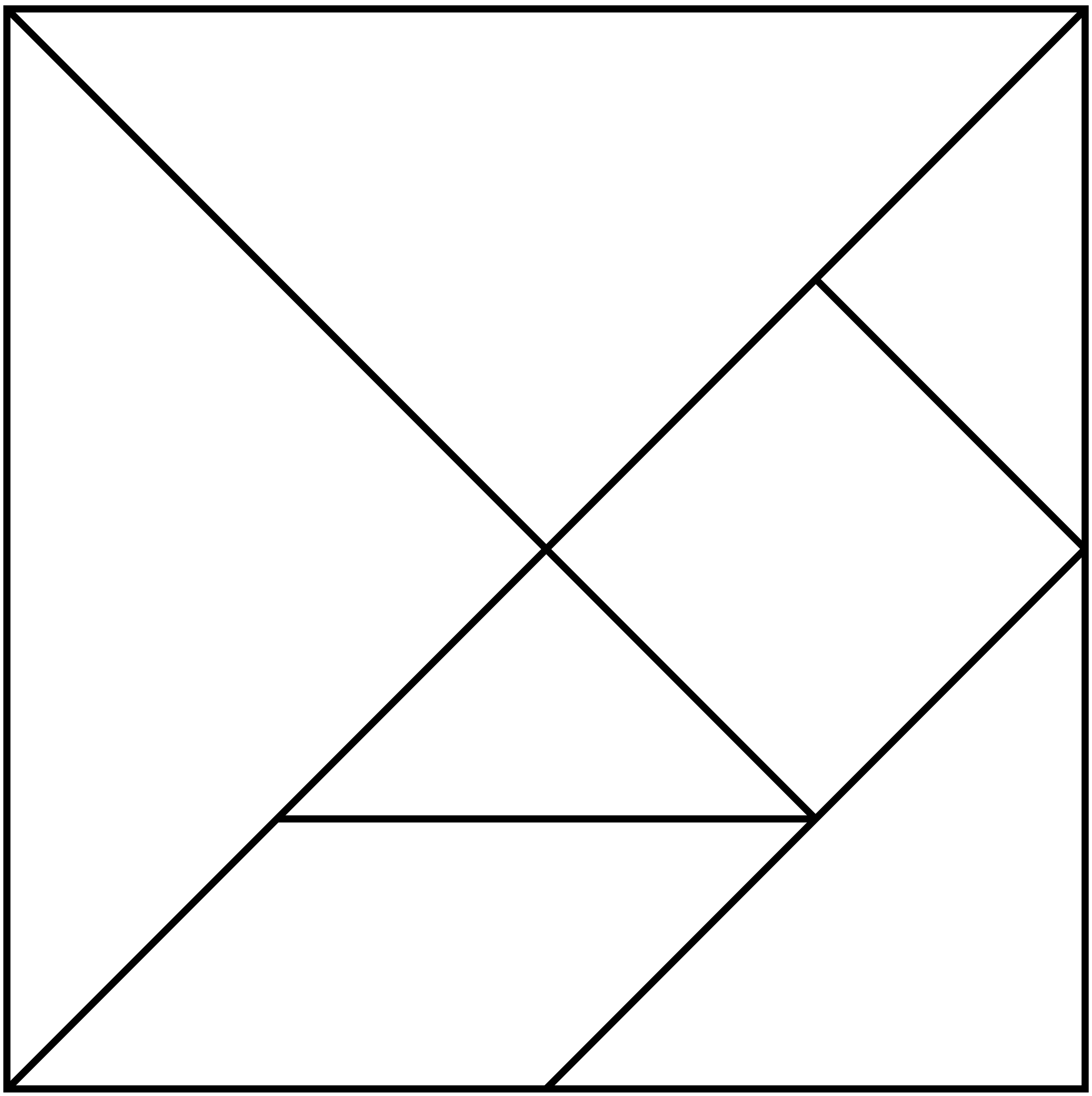
Draw a one inch (three cm.) grid on the material to make the puzzle. You then mark off the blue lines as shown below. Cut your material carefully along these blue lines. This will produce the seven tan pieces; five triangles, one square and one rhomboid. Make sure to cut precisely.



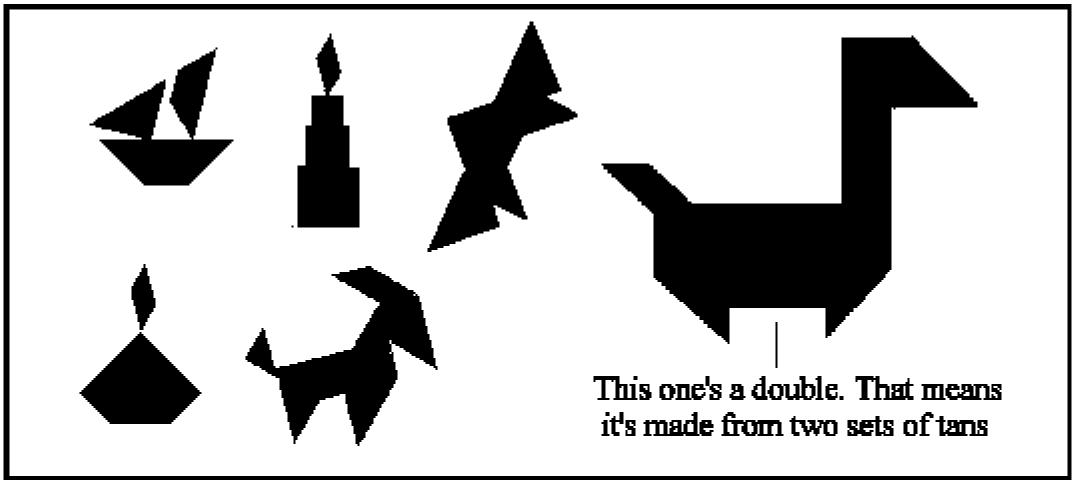
Tangram Template



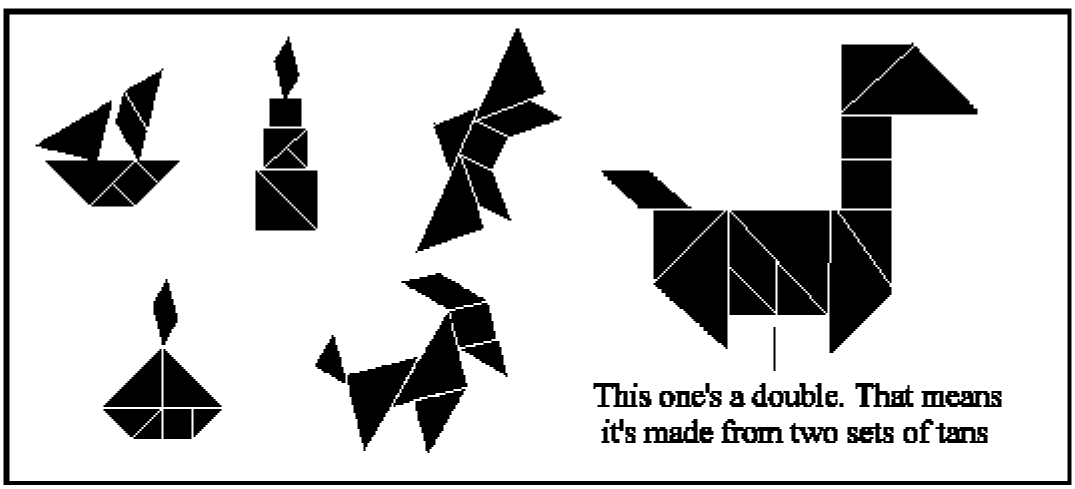
Tangram Template



Sample Tangram Puzzles



Tangram Answers



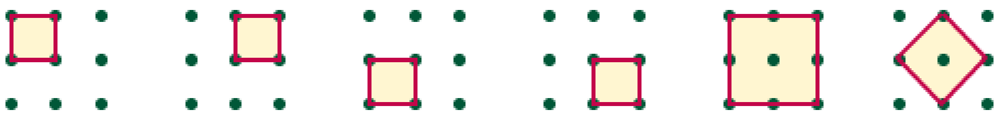
Tangrams. Randy Send. Retrieved from the World Wide Web at: http://tangrams.ca/index.htm.

Geoboards

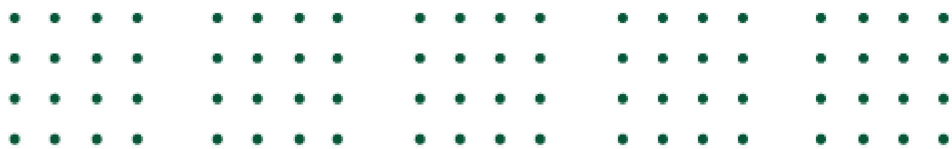
A geoboard assists in the teaching of basic geometric concepts. Use geoboards to illustrate area, perimeter, and rational number concepts. Geoboards develop problem solving and teach patterning, perimeter, symmetry and more.

A fascinating initial problem on the geoboard is to determine how many squares can be constructed. It is necessary to consider all possible sizes and all possible positions. Squares may be made on a geoboard so that the base of the square either is or is not parallel to the base of the geoboard. Students may wish to see the differences between the size of the geoboard and the number of squares that they can locate.

A 2 by 2 geoboard can have only one square, and this square is parallel to the base of the geoboard. A 3 by 3 geoboard can have 6 squares in all. Five of these squares are parallel to the base of the geoboard. There are 3 different squares shown of the 6 possible. The squares are shown below:

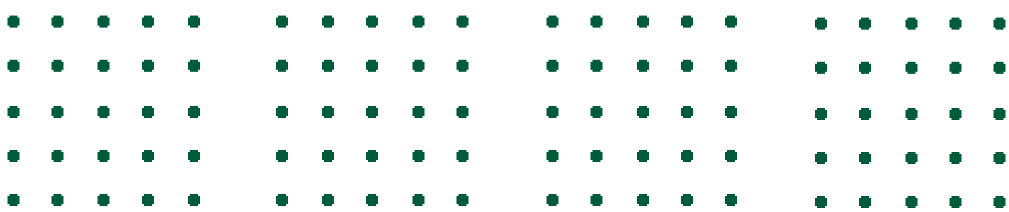


Have students find all the squares that can be drawn on a 4 by 4 geoboard. Begin by finding all the different squares that are parallel to the base and the number of each size. Next, find all the squares that are not parallel to the base and the number of each. Have students draw the different sizes of squares and record the number of each below the dot grid.

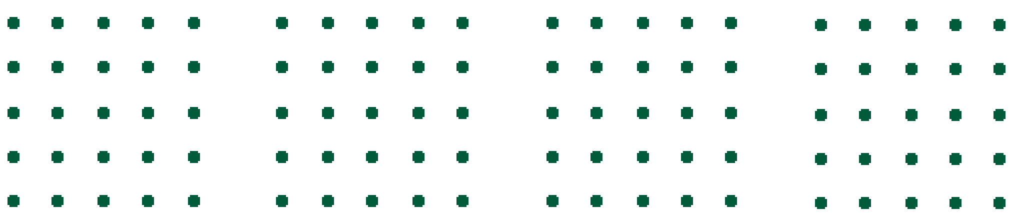


Total = ____ Total = ____ Total = ____ Total = ____ Total = ____

Have students find all the squares that can be drawn on a 5 by 5 geoboard. Begin by finding all the different squares that are parallel to the base and the number of each size. Next, find all the squares that are not parallel to the base and the number of each. Have students draw the different sizes of squares and record the number of each below the dot grid.



Total = _____ Total = _____ Total = _____ Total = _____



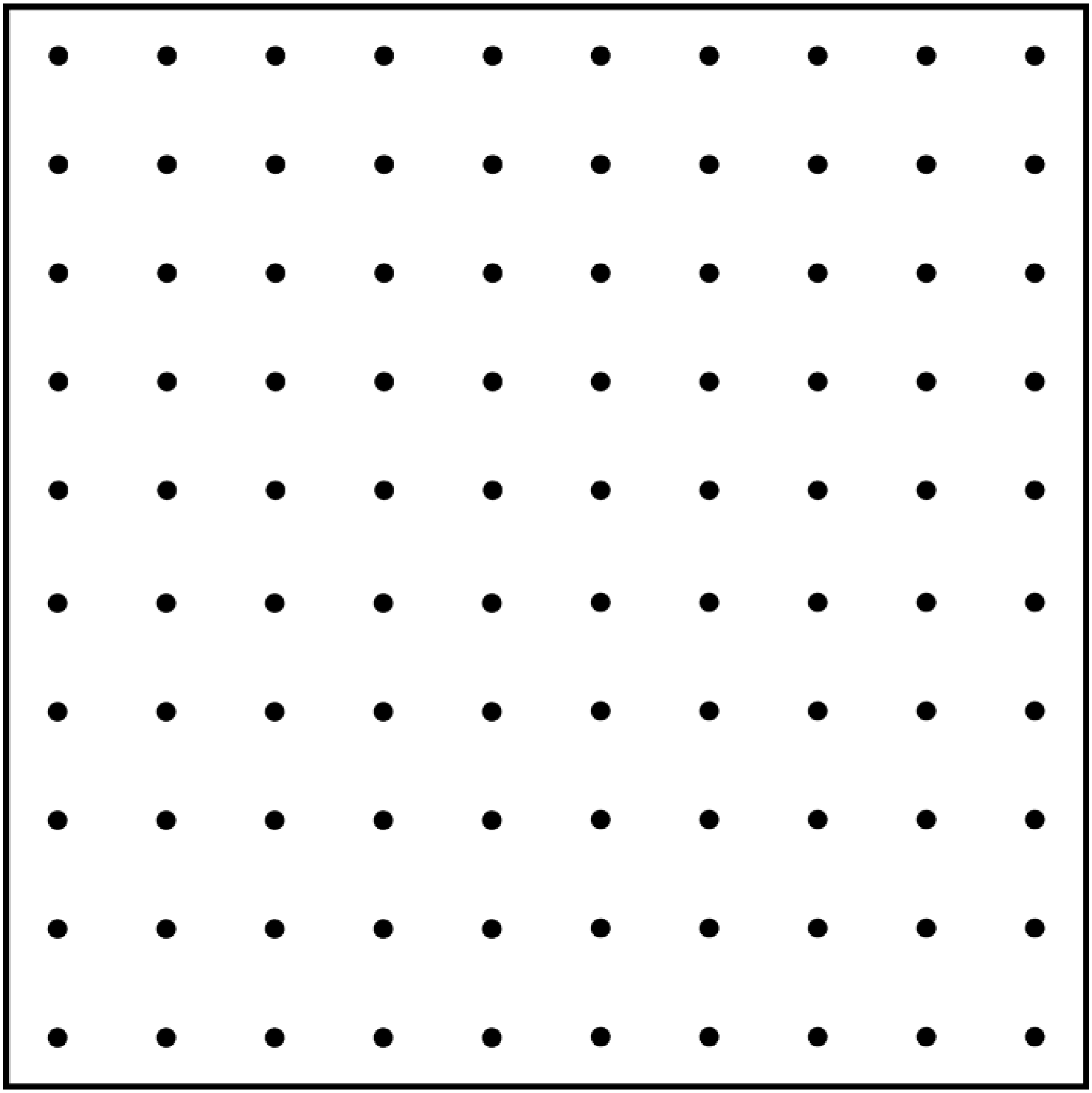
Total = _____ Total = _____ Total = _____ Total = _____

Have students use the results of their investigations to complete a table similar to the one below. Have them use the patterns in the table to extend the results to a 6 by 6, 7 by 7, 8 by 8, 9 by 9, and 10 by 10 geoboard.

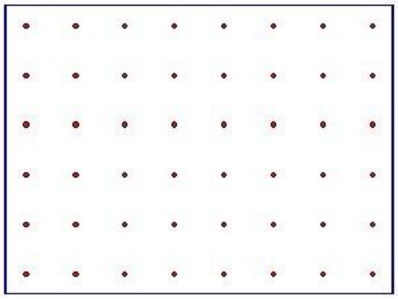
	Parallel to Base		Not Parallel to Base		
Size	**Different**	**Total**	**Different**	**Total**	**Grand Total**
1 by 1	**0**	**0**	**0**	**0**	**0**
2 by 2	**1**	**1**	**0**	**0**	**1**
3 by 3	**2**	**5**	**1**	**1**	**6**
4 by 4					
5 by 5					
6 by 6					
7 by 7					
8 by 8					
9 by 9					
10 by 10					

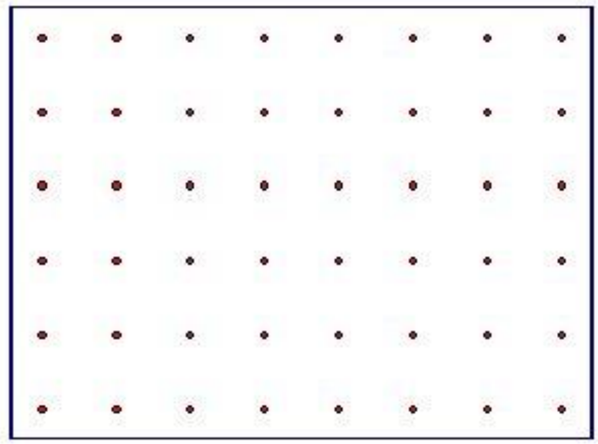
Geoboards are an excellent way to teach geometric skills, including area and perimeter.

Geoboard Template

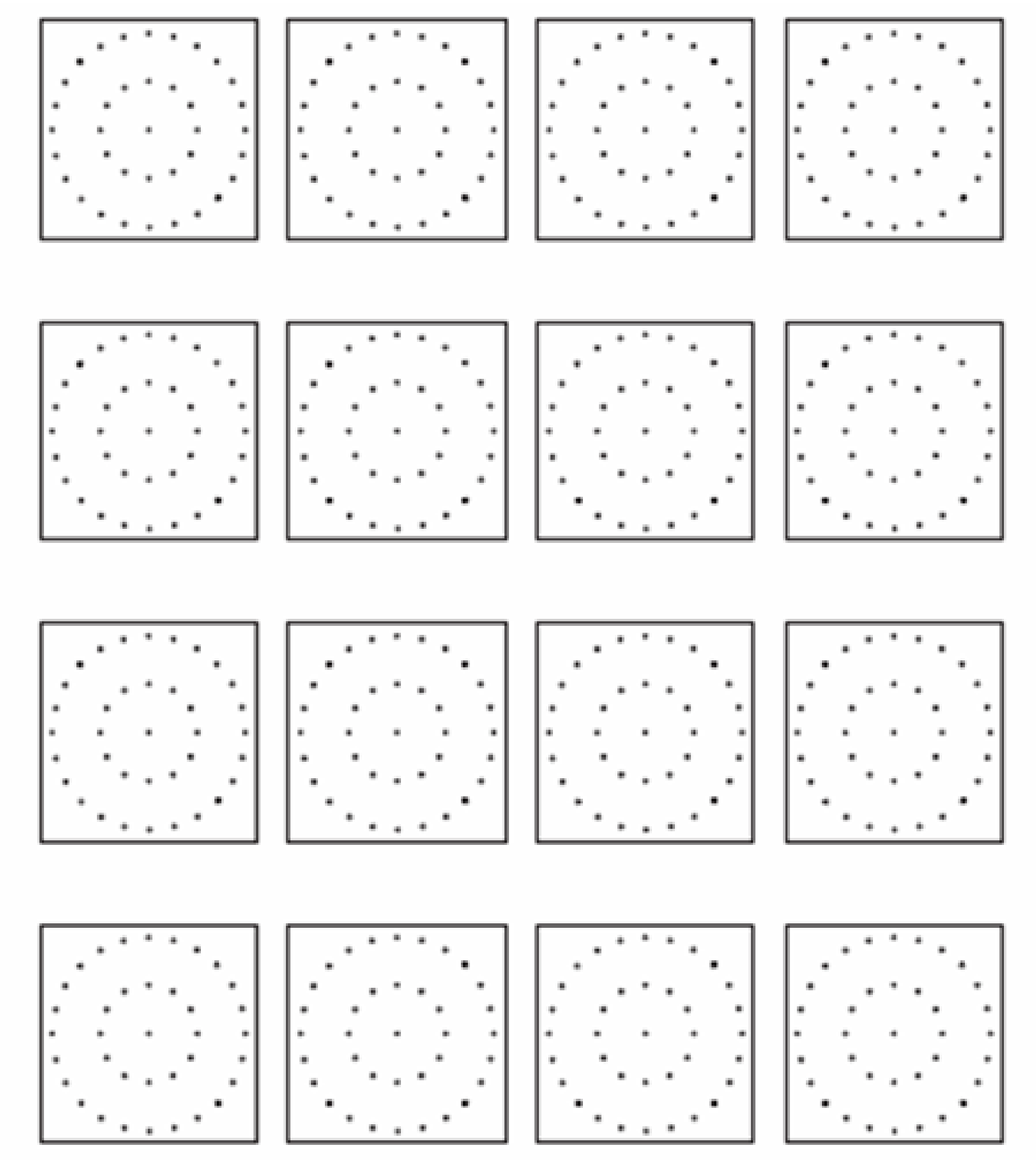


Geoboard Templates





Circular Geoboard Template



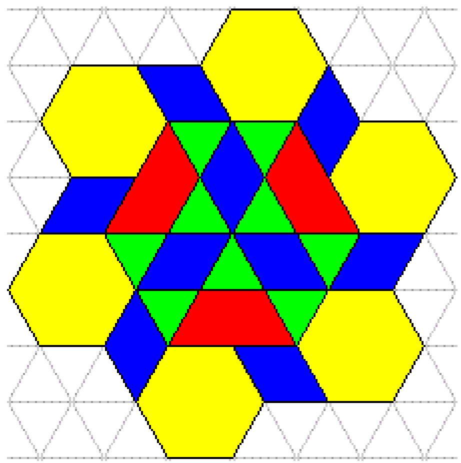
Pattern Blocks

Pattern blocks are useful to assist students in exploring fractions and percents, as well as to learn about geometric figures and patterns.

A sample activity is as follows:

Create a design with mirror or rotation symmetry using triangles, rhombuses, trapezoids, and/or hexagons. Let a triangle have a value of 1. Make a chart to show the value of each shape:

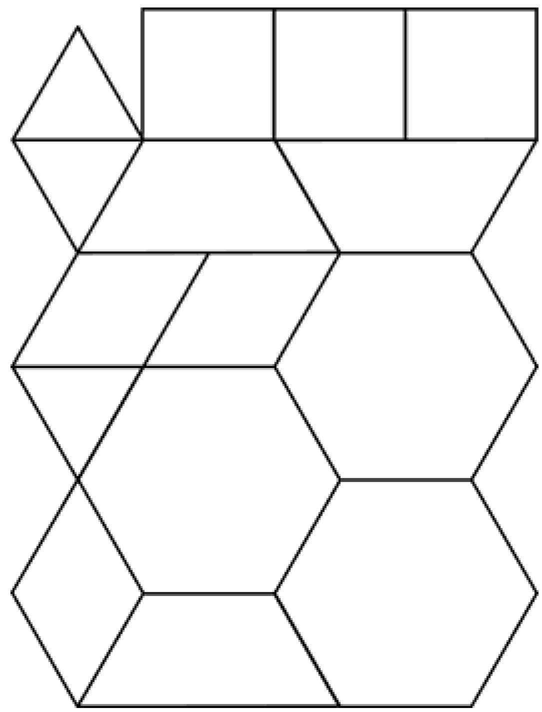
Example:



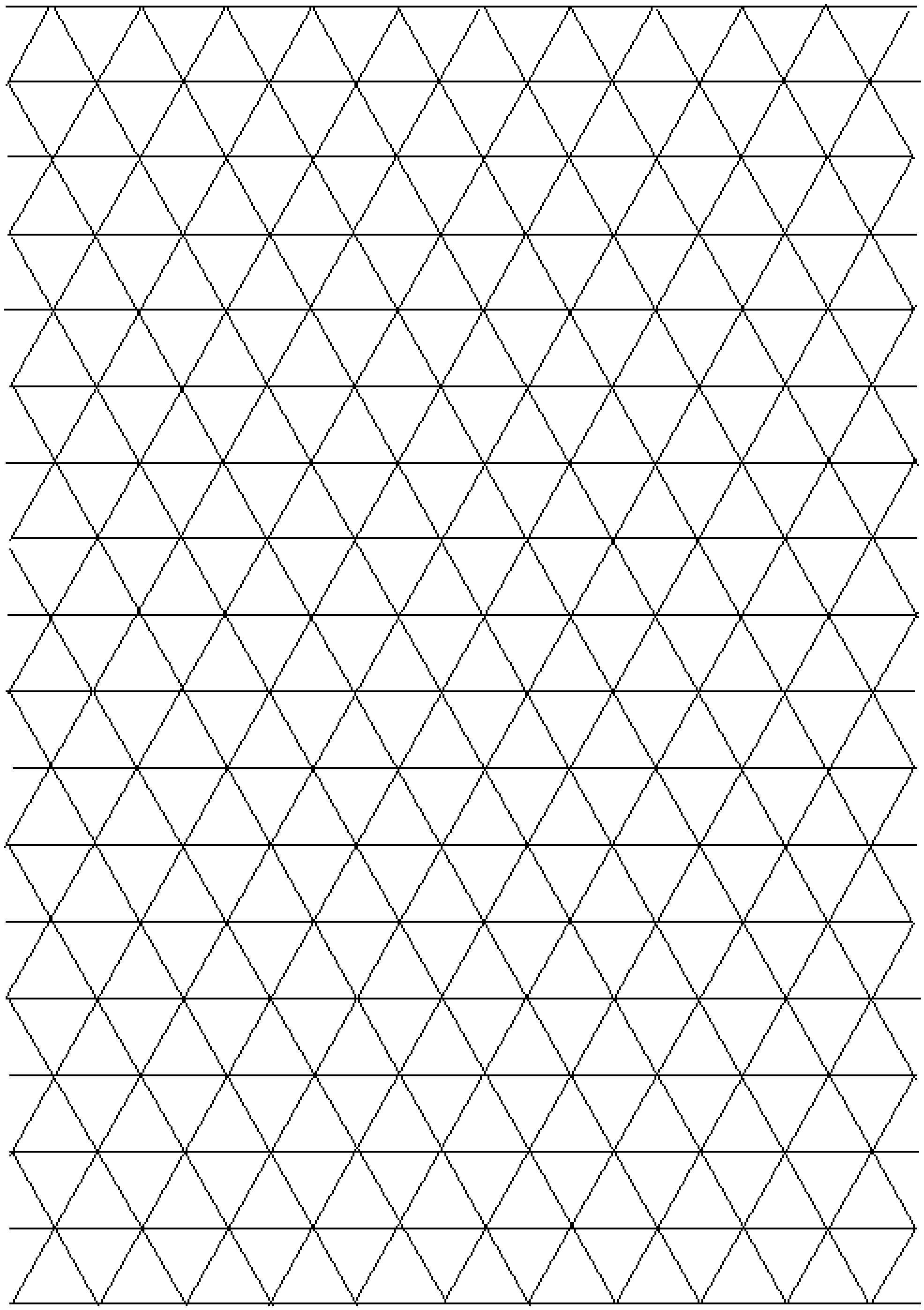
	Value of 1 Block	Number of Blocks Used	Total Value in My Design	Fraction of My Design	Percentage of My Design
Triangle	1	9	9	1/8	12.5%
Rhombus	2	9	18	¼	25%
Trapezoid	3	3	9	1/8	12.5%
Hexagon	6	6	36	½	50%
Total, All Blocks	Varies	27	72	1	100%

Smaller Pattern Block Template (Triangle, Square, and Smaller Rombus)

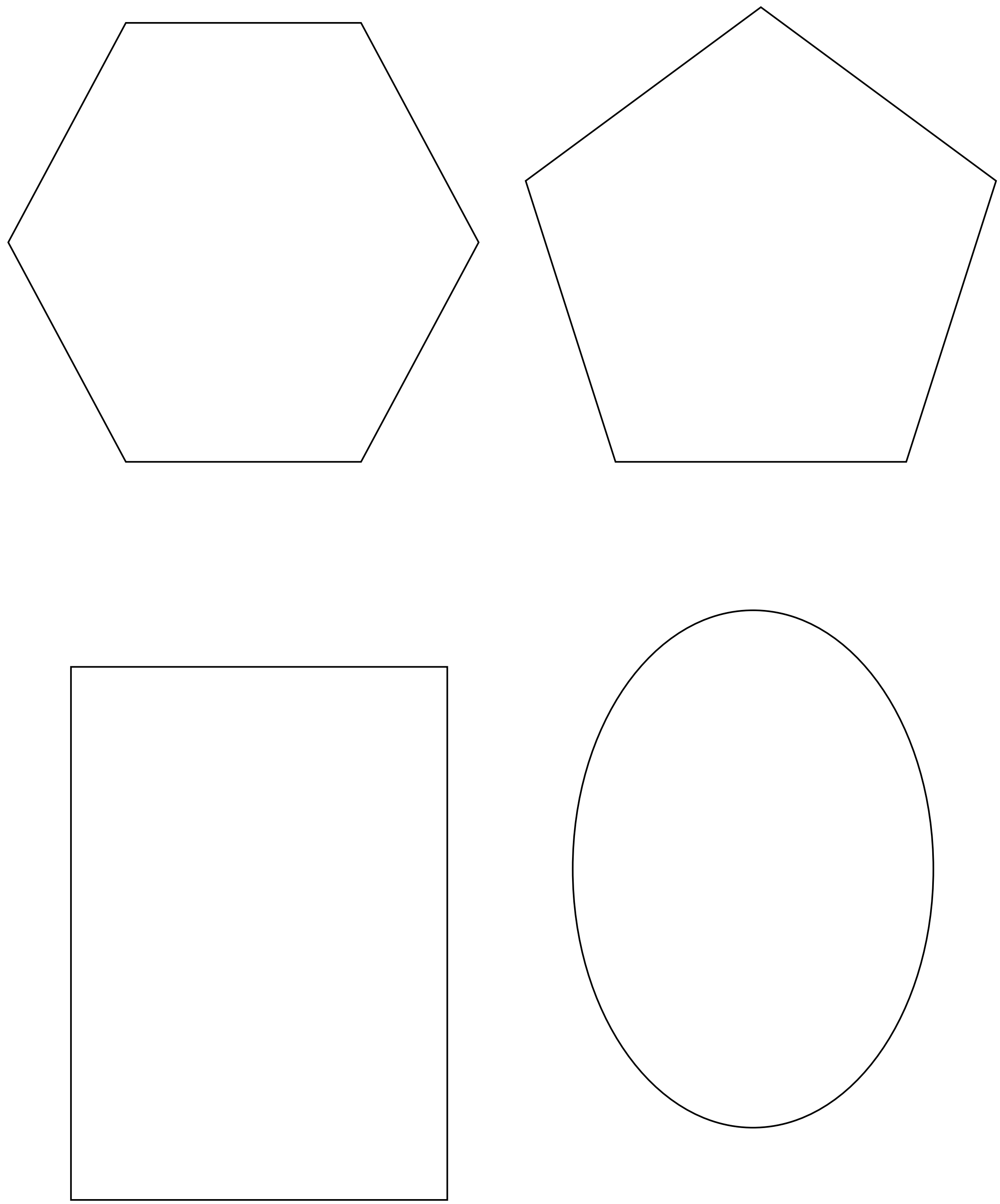
Cut along the solid lines. In smaller pattern block sets, a triangle is green, a square is orange and the smaller rhombus is beige in color.

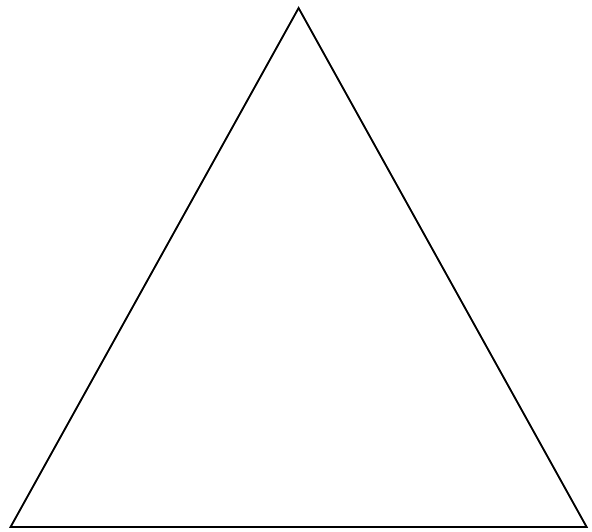


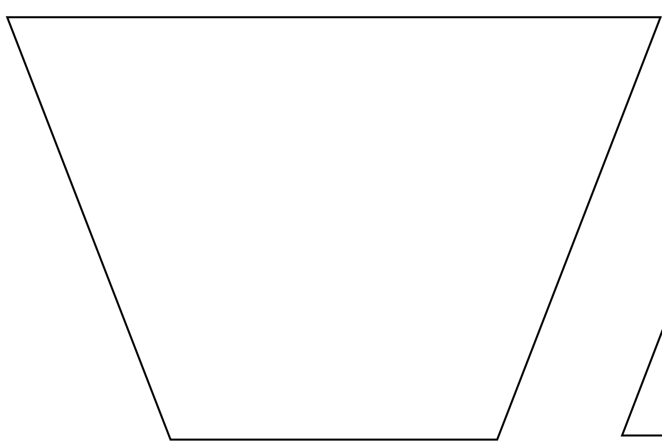
Pattern Block Grid

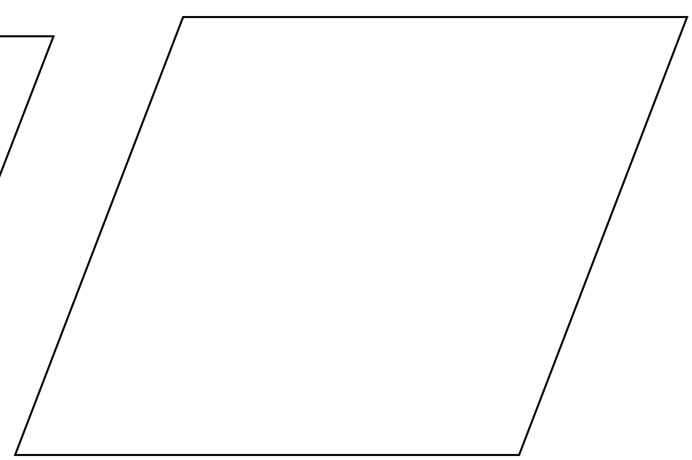


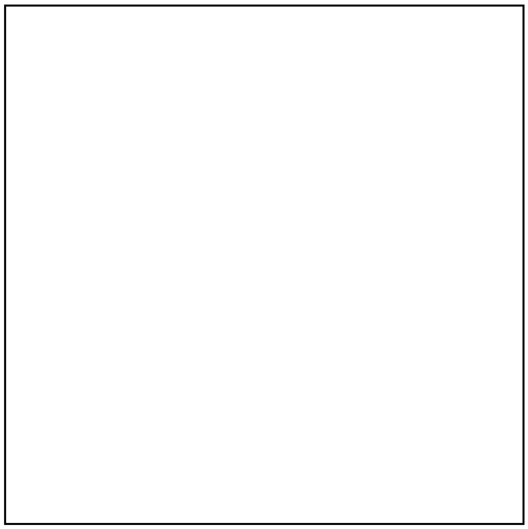
Larger Set of Pattern Block Template

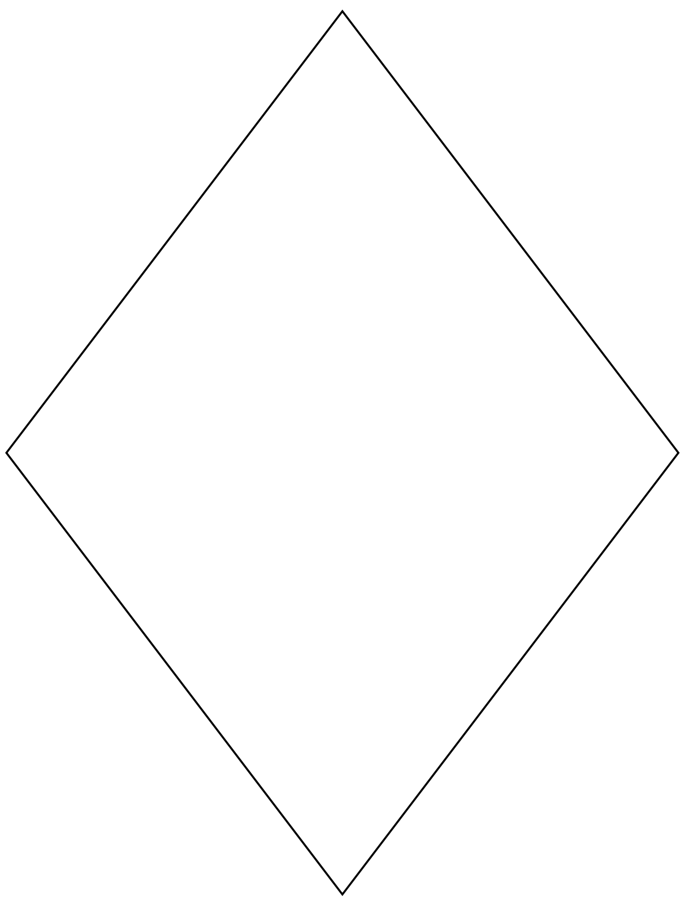




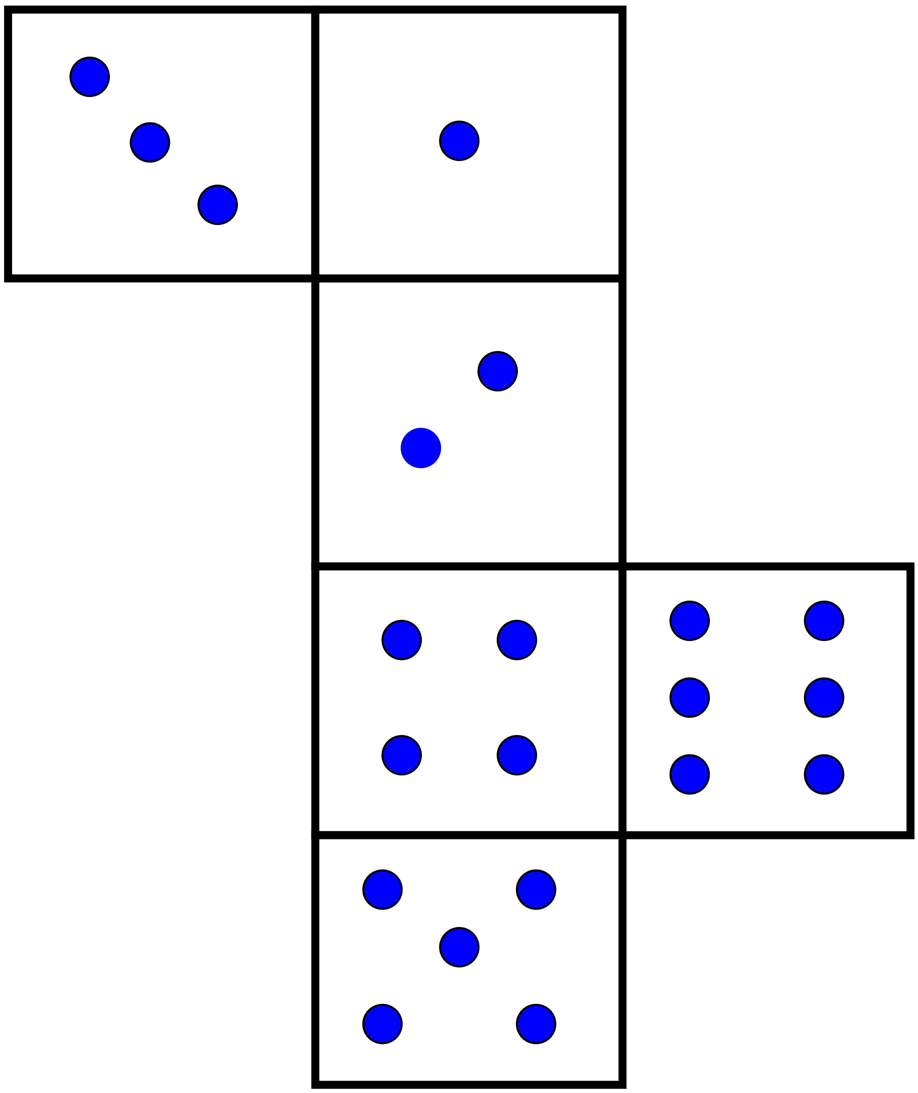






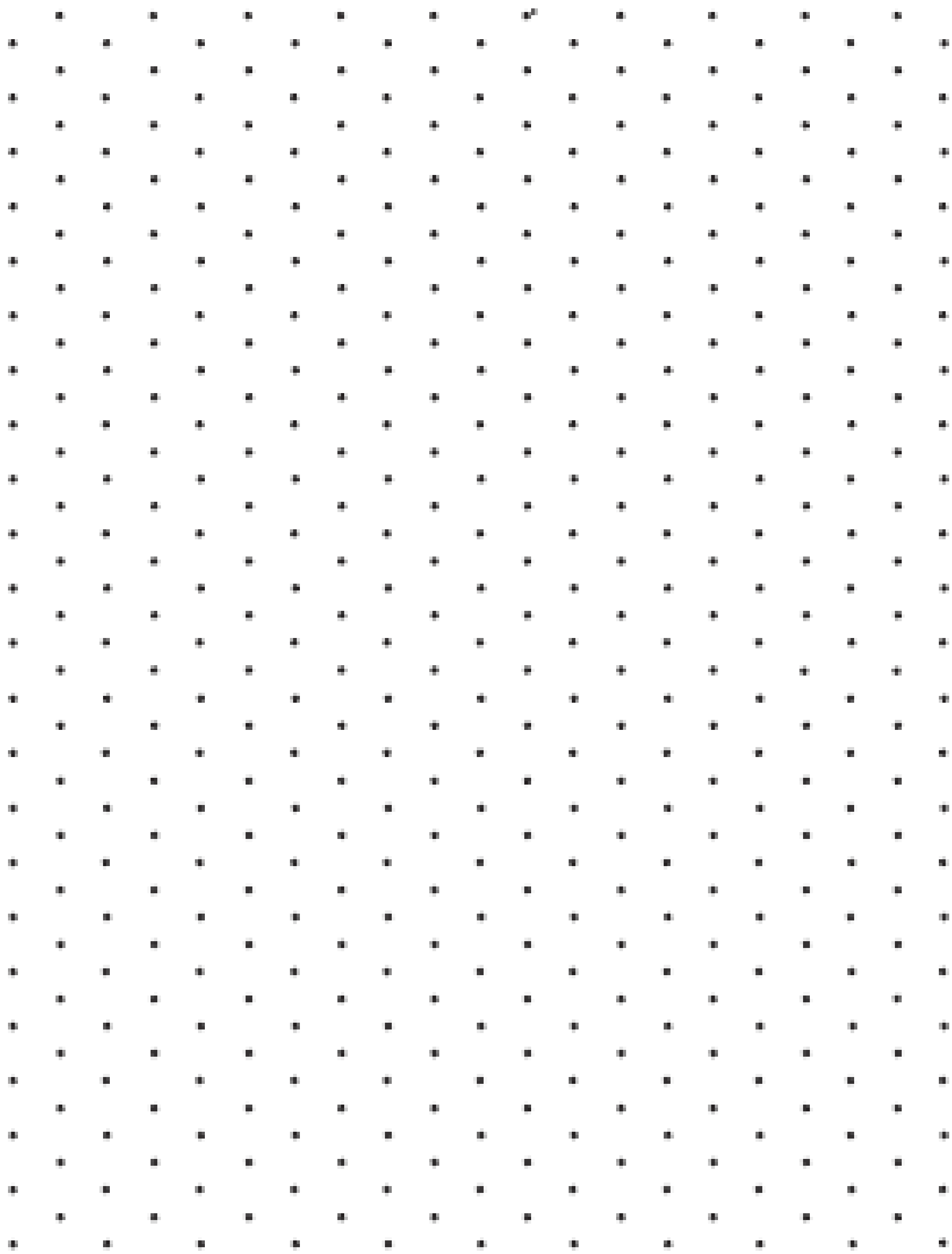


Dice Template



Graph Paper

Isometric Dot Template



Algebra Tiles