Math DefragGED: Problem Solving

Webinar
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Handout

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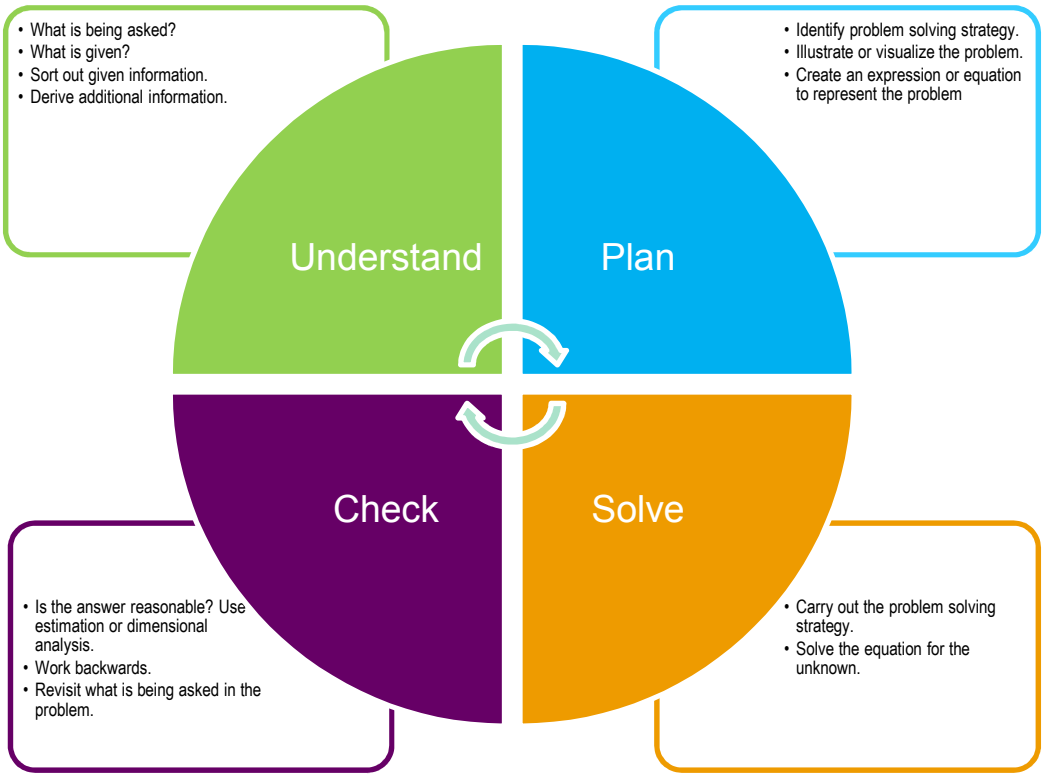
Polya's 4 Step Problem Solving Approach



Mathematical Practices

Mathematical Practices	Range of Depth of Knowledge
MP.1 Building Solution Pathways and Lines of Reasoning	
• Search for and recognize entry points for solving a problem.	1-2
• Plan a solution pathway or outline a line of reasoning.	1-3
• Select the best solution pathway, according to given criteria.	2-3
• Recognize and identify missing information that is required to solve a problem.	1-2
• Select the appropriate mathematical technique(s) to use in solving a problem or a line of reasoning.	1-3

Mathematical Practices	Range of Depth of Knowledge
MP.2 Abstracting Problems • Represent real world problems algebraically. • Represent real world problems visually. • Recognize the important and salient attributes of a problem.	1-2 1-2 2-3
MP.3 Furthering Lines of Reasoning • Build steps of a line of reasoning or solution pathway, based on previous step or givens. • Complete the lines of reasoning of others. • Improve or correct a flawed line of reasoning.	1-3 1-3 2-3
MP.4 Mathematical Fluency • Manipulate and solve arithmetic expressions. • Transform and solve algebraic expressions. • Display data or algebraic expressions graphically.	1-2 1-2 1-2
MP.5 Evaluating Reasoning and Solution Pathways • Recognize flaws in others' reasoning. • Recognize and use counterexamples. • Identify the information required to evaluate a line of reasoning.	2-3 2-3 2-3

Common Problem Solving Techniques

- Illustrate the Problem
- Guess and Check
- Simplify the Problem
- Look for Patterns
- Break Problem to Smaller Pieces
- Work Backwards

Sample Word Problems

1. During a catering event, a chef is trying to place as many serving dishes on a buffet table as possible. What is the maximum number of circular serving dishes that the chef can fit on a 4 x 8 foot rectangular table, if all serving dishes have a radius of 6 inches? Use π=3.14.

   ○ A. 8

   ○ B. 10

   ○ C. 32

   ○ D. 128

Sample Word Problems

2. At 10:30AM, the track and field coach asked Taylor and Damien to start warming up by running around the track. Taylor knew he can run at an average rate of 1.5 miles per hour faster than Damien so he gave him a 30 minute head start. If Damien runs at an average rate of 3.5 miles per hour, complete the prediction below.

Taylor will catch up with Damien after Select... at around Select...

Select...
1hr., 10mins.
1hr., 16mins.
1hr., 40mins.
2hrs., 7mins.

Select...
11:40 AM
11:46 AM
12:10 PM
12:37 PM

Sample Word Problems

3. Fruti-Yo, a self-serve yogurt store, sells their yogurt by the ounce, plus a $0.05 charge for the cup. Table 1, shows different customers, weights of yogurt and the corresponding prices. Write an equation to describe the total cost of any purchase of yogurt at Fruti-Yo?

Table 1

Customer	Weight	Price
Harold	8.2	$ 3.28
Ophelia	10.5	$ 4.20
Aaron	9.6	$ 3.84

Answer:

Sample Word Problems

4. If Eric fills his yogurt cup up to 15oz, how much would he pay at the counter?

   Click on the price you want to select and drag it into the corresponding section of Table 1.

$ 5.45	$ 6.05	$ 13.13	$ 13.55

Table 1

Customer	Weight	Price
Harold	8.2	$ 3.28
Ophelia	10.5	$ 4.20
Aaron	9.6	$ 3.84
Eric	15.0	

Answer Key to Sample Word Problems

1. C
2. Taylor will catch up with Damien after 1 hr, 10mins. at around 11:40AM.
3. $y = 0.4x + 0.05$
4. $ 6.05

Additional Practice Word Problems

1. The sum of the ages of Ron and Harry is 48 years. Eight years from now, Harry will be three times Ron's age. How old is Ron?

2. During the One Direction concert, each reserved seat costs $150.00 and the rest of the seats were $100.00 each. If 3,200 tickets were sold and the total proceeds of the concert was $ 350,000.00, how many reserved tickets were sold?

3. Eight boys rented a speed boat for their trip from Ft. Meyers to Key West, splitting the cost of the rental equally. Three of the boys backed out at the last minute. As a result, each of the remaining five boys had to pay $ 90.00 more than they initially planned. How much was the total cost of renting the speed boat?

Solutions to Additional Practice Word Problems

1. The sum of the ages of Ron and Harry is 48 years. Eight years from now, Harry will be three times Ron's age. How old is Ron?

   R = Ron's age
   H = Harry's age

   First Equation: $R + H = 48$
   Second Equation: $3(R + 8) = H + 8$

   Solving for H in the first equation:
   $H = 48 - R$

   Substituting this resulting equation to the second equation:
   $3(R + 8) = (48 - R) + 8$
   $3R + 24 = 48 - R + 8$
   $3R + 24 = -R + 56$
   $3R + R = 56 - 24$
   $4R = 32$
   $R = 8$

   Ron is 8 years old.

Solutions to Additional Practice Word Problems

2. During the One Direction concert, each reserved seat costs $150.00 and the rest of the seats were $100.00 each. If 3,200 seats were sold and the total proceeds from the concert was $ 350,000.00, how many reserved seats were sold?

   x = Number of reserved seats
   y = Number of regular seats

   First Equation: $150x + 100y = 350000$
   Second Equation: $x + y = 3200$

   Solving the second equation for x:
   $x = 3200 - y$

   Substituting this resulting equation to the first equation:
   $150(3200 - y) + 100y = 350000$
   $480000 - 150y + 100y = 350000$
   $-50y = 350000 - 480000$
   $-50y = -130000$
   $y = 2600$

   Knowing that there are 2600 regular seats, we can use the second equation to determine the number of reserved seats.
   $x + 2600 = 3200$
   $x = 600$

   There were 600 reserved seats sold.

Solutions to Additional Practice Word Problems

3. Eight boys rented a speed boat for their trip from Ft. Meyers to Key West, splitting the cost of the rental equally. Three of the boys backed out at the last minute. As a result, each of the remaining five boys had to pay $ 90.00 more than they initially planned. How much was the total cost of renting the speed boat?

   c = cost per person
   r = rental cost

   Initially:
   $8c = r$ (8 people splitting the rental cost)

   After 3 boys backed out:
   $5(c + 90) = r$ (5 people splitting the rental cost)

   Because the two equations above equal the same amount of rental cost:
   $8c = 5(c + 90)$

   Solving for c:
   $8c = 5c + 450$
   $3c = 450$
   $c = 150$

   This means the initial cost per person is $150.00. We can use this to determine the speed boat rental cost = 8c.
   $8(150) = 1200$

   The rental cost for the speed boat is $ 1,200.00.

What skills are essential to problem solving?

- Differentiating between essential and non-essential information
- Attention to detail
- Abstracting word problems and vice versa
- Illustrating word problems (i.e. diagrams, tables, or graphs)
- Recognizing patterns
- Analyzing data
- Estimation

How do we decrease student anxiety over word problems?

- Introduce word problems as soon as possible.
- Use word problems that are relevant to students
- When developing computational or procedural fluency, use real-world analogies or representations
- Allow students to explore problems before immediately rushing to the algebraic representations.
- Teach students how to illustrate word problems or use manipulatives when appropriate.
- At the end of the problem solving process, train students to write a sentence relating the answer to the question.

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