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INSTITUTE FOR THE PROFESSIONAL
DEVELOPMENT OF ADULT EDUCATORS

Putting Manipulatives to Work

October 4, 2017

www.floridaipdae.org

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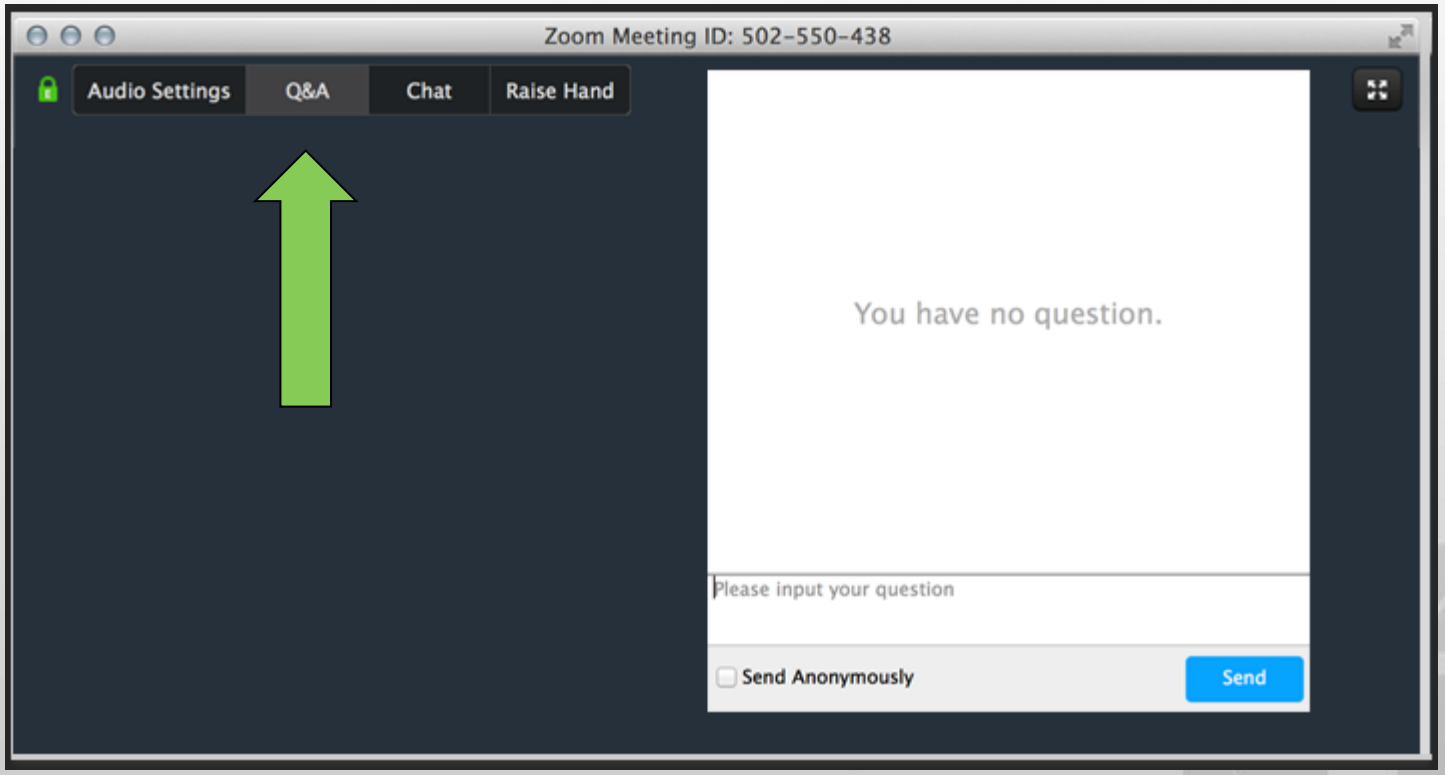
Webinar Things to Remember

- If you have a question, please type it into the **Q&A** option.



- Attendee microphones will be muted. You will be in **listen only** mode.
- Today's presentation is being **recorded**. It will be archived and available on the IPDAE website within 48 hours.

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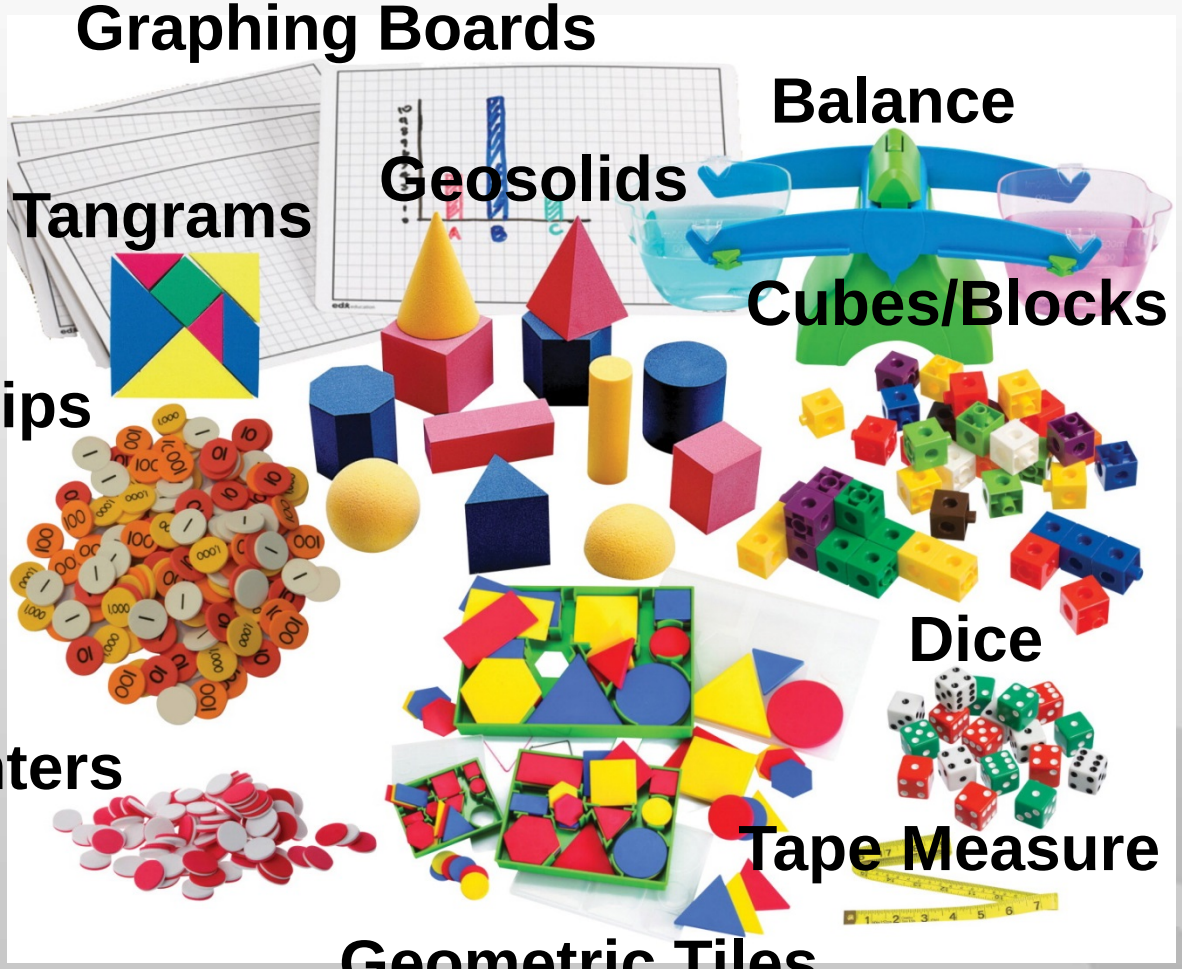
Agenda

I. What are manipulatives?
II. Different Types of Manipulatives
III. Why Use Manipulatives?
IV. When to Use Manipulatives?
V. Manipulatives in the Standards and Frameworks
VI. Some Demonstrations on the Use of Virtual Manipulatives in Mathematics
VII. IPDAE Resources on Manipulatives
VIII. Key Ideas When Using Manipulatives
IX. Q&A
X. Evaluation

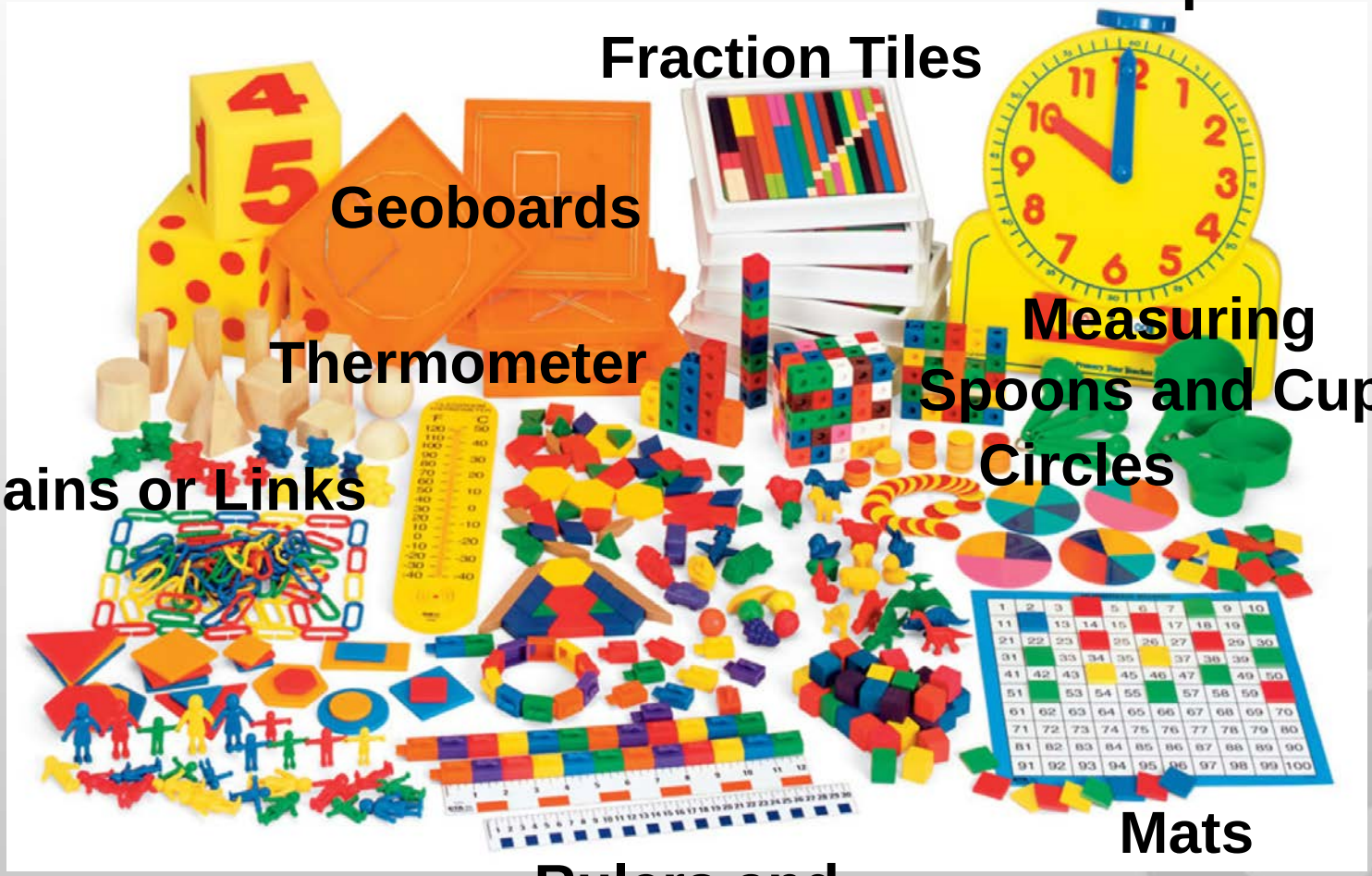
What are Manipulatives?

Manipulatives are any concrete object or tools used for instruction.

Some Examples



More Examples



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Virtual Manipulatives


Fraction Pieces – Work with parts and wholes to learn about fractions.


Fractions - Adding – Illustrates what it means to find a common denominator and combine.


Fractions - Comparing – Judge the size of fractions and plot them on a number line.


Fractions - Equivalent – Illustrates relationships between equivalent fractions.


Fractions - Rectangle Multiplication – Visualize and practice multiplying fractions using an area representation.


Function Machine – Explore the concept of functions by putting values into this machine and observing its output.


Golden Rectangle – Illustrates iterations of the Golden Section.


Grapher – A tool for graphing and exploring functions.


Mastermind – Use inference and logic to play a game and guess a hidden pattern of pegs.


Money – Learn about money by counting and making change.


Number Line Bars - Fractions – Divide fractions using number line bars.

Fractions

Open Web App
Apple App Store
Chrome Store

The Fractions app lets students use a bar or circle to represent, compare, and perform operations with fractions with denominators from 1 to 100. Choose the fraction model and number of equal parts. Use a color to select specific parts to show a fraction of the whole.

Geoboard

Open Web App
Apple App Store
Windows Store
Chrome Store

The Geoboard app is a tool for exploring a variety of mathematical topics introduced in the elementary and middle grades. Learners stretch bands around the pegs to form line segments and polygons and make discoveries about perimeter, area, angles, congruence, fractions, and more.

Math Vocabulary Cards

Open Web App
Apple App Store
Windows Store
Chrome Store

Math Vocabulary Cards help students deepen their conceptual understanding of key terms in mathematics. Each card features three sections: a math term, a representative example or model, and a concise definition.

Money Pieces

Open Web App
Apple App Store
Chrome Store

Money Pieces help students visualize and understand money values and relationships. Two versions of coins and bills are provided: virtual currency pieces that replicate the appearance and relative size of U.S. coins and the dollar bill, and area money pieces.

Number Frames

Open Web App
Apple App Store
Windows Store
Chrome Store

Number Frames help students structure numbers to 5, 10, 20, and 100. Students use the frames to count, represent, compare, and compute with numbers in a particular range.

Number Line

Open Web App
Apple App Store
Windows Store
Chrome Store

Number Line helps students visualize number sequences and illustrate strategies for counting, comparing, adding, subtracting, multiplying, and dividing. Choose number lines labelled with whole numbers, fractions, decimals, or negative numbers.

Number Pieces

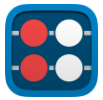
Open Web App
Apple App Store
Windows Store
Chrome Store

Number Pieces helps students develop a deeper understanding of place value while building their computation skills with multi-digit numbers. Students use the pieces to represent multi-digit numbers, regroup, add, subtract, multiply, and divide.

Number Pieces Basic

Open Web App
Apple App Store

Number Pieces Basic is a simplified version of Number Pieces. It has fewer features, putting greater focus on place value, counting, addition, and subtraction with multi-digit numbers.



Number Rack

Open Web App
Apple App Store
Windows Store
Chrome Store

Number Rack facilitates the natural development of children's number sense. Rows of movable, colored beads encourage learners to think in groups of fives and tens, helping them to explore and discover a variety of addition and subtraction strategies. Free activities and free book available.

Pattern Shapes

Open Web App
Apple App Store
Windows Store
Chrome Store

Students use Pattern Shapes to explore geometry and fractions, create their own designs, or filling in outlines. As they work with shapes, students think about angles, investigate symmetry, and compose and decompose larger shapes.

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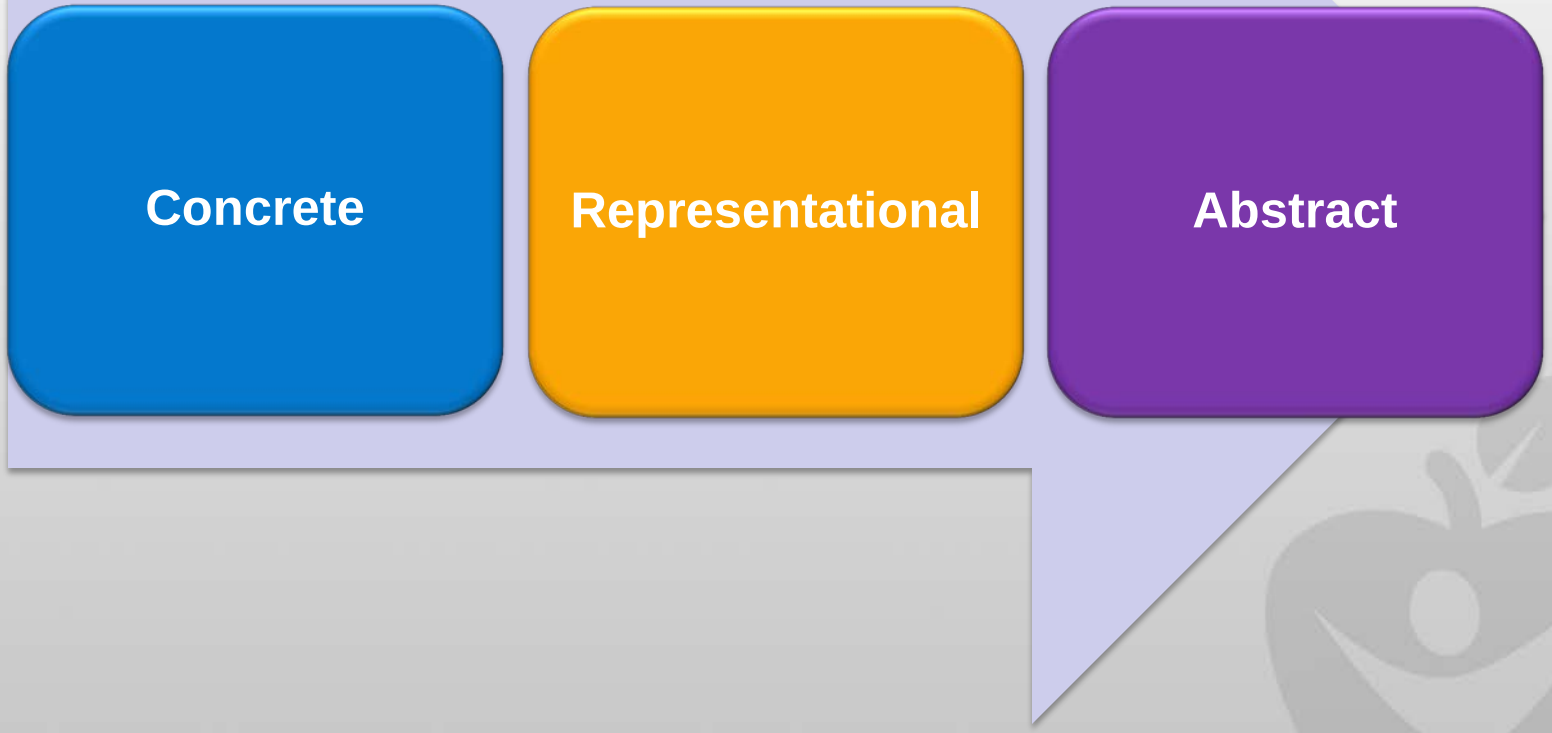
Why Use Manipulatives?

- Mathematics is the lowest performing area among the 4 major subject areas in Adult Basic Education and GED.
- Mathematics reasoning spans other subject areas such as science and social studies.
- One of the greatest challenges for students is visualizing word problems in a way that would lead them to a strategy towards a solution.
- There are so many abstract concepts in mathematics that students do not understand. Students need to understand the abstract concepts before they can apply steps and processes with fidelity.
- Not all students learn the same way. Manipulatives give us as way to further differentiate instruction.

When and how do you use manipulatives in your classroom?

Experiential Learning



(Hartshorn and Boren, 1990; Heddens, 1986; Reisman,1982; Ross and Kurtz, 1993).

When to Use Manipulatives?

- What is your experience in the use of manipulatives?
- How did you use manipulatives? What topics or skills did you teach using manipulatives?
- How did your students respond with using manipulatives?

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When to Use Manipulatives?

The NCTM calls for manipulatives to be used in teaching a wide variety of topics in mathematics:

- **sorting** — a pre-mathematical skill that aids in comprehension of patterns and functions
- **ordering** — a pre-mathematical skill that enhances number sense and other math-related abilities
- **distinguishing patterns** — the foundation for making mathematical generalizations
- **recognizing geometric shapes** and understanding relationships among them
- **making measurements**, using both non-standard and standard units with application to both two and three-dimensional objects

National Council of Teachers of Mathematics (2000 – present).

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When to Use Manipulatives?

- understanding the **base-ten system of numbers**
- comprehending **mathematical operations** — addition, subtraction, multiplication, division
- recognizing **relationships among mathematical operations**
- exploring and describing **spatial relationships**
- identifying and describing different types of **symmetry**
- developing and utilizing **spatial memory**
- learning about and experimenting with **transformations**
- engaging in **problem-solving**
- **representing mathematical ideas** in a variety of ways
- **connecting different concepts** in mathematics
- **communicating mathematical ideas effectively**

National Council of Teachers of Mathematics (2000 – present).

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Manipulatives in the Standards

Standards for Mathematical Practice

Make sense of problems and persevere in solv

Mathematically proficient students start by explainin
looking for entry points to its solution. They analyze
make conjectures about the form and meaning of the
simply jumping into a solution attempt. They consid
simpler forms of the original problem in order to gai
evaluate their progress and change course if necessa
problem, transform algebraic expressions or change
to get the information they need. Mathematically pr
between equations, verbal descriptions, tables, and g
relationships, graph data, and search for regularity o
using concrete objects or pictures to help conceptual
students check their answers to problems using a dif
themselves, "Does this make sense?" They can unde
problems and identify correspondences between diff

Reason abstractly and quantitatively. (MP.2)

Mathematically proficient students make sense of qu
situations. They bring two complementary abilities t
relationships: the ability to *decontextualize*—to abst
and manipulate the representing symbols as if they h
attending to their referents—and the ability to *conte*
manipulation process in order to probe into the refer
reasoning entails habits of creating a coherent repres
units involved; attending to the meaning of quantitie
flexibly using different properties of operations and

Construct viable arguments and critique the r

Mathematically proficient students understand and u
established results in constructing arguments. They r
statements to explore the truth of their conjectures. T
them into cases, and can recognize and use counterex
communicate them to others, and respond to the argu
data, making plausible arguments that take into acco
Mathematically proficient students are also able to c
arguments, distinguish correct logic or reasoning fro
an argument—explain what it is. Less experienced st
referents such as objects, drawings, diagrams, and ac
correct, even though they are not generalized or mad

determine domains to which an argume
of others, decide whether they make se
arguments.

Model with mathematics. (MP.4)

Mathematically proficient students can
everyday life, society, and the workpla
describe a situation. A student might ap
problem in the community. A student n
to describe how one quantity of interes
can apply what they know are comfort
complicated situation, realizing that the
quantities in a practical situation and m
tables, graphs, flowcharts and formulas
conclusions. They routinely interpret th
reflect on whether the results make sen

Use appropriate tools strategically

Mathematically proficient students con
These tools might include pencil and p
spreadsheet, a computer algebra systen
Proficient students are sufficiently fam
decisions about when each of these too
and their limitations. For example, mat
solutions generated using a graphing ca
estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Attend to precision. (MP.6)

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. Less experienced students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Look for and make use of structure. (MP.7)

Mathematically proficient students look closely to discern a pattern or structure. Students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Look for and express regularity in repeated reasoning. (MP.8)

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Early on, students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2+x+1)$, and $(x-1)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

IPDAE (2016). College and Career Readiness Standards Workshop Part 2. www.floridaipdae.org – ABE – Workshops

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Manipulatives in the Curriculum Frameworks

ADULT BASIC EDUCATION MATHEMATIC DOMAINS					
Domain Number	NRS Reporting Grade Equivalent (GE)	NRS Level 1 0.0 – 1.9	NRS Level 2 2.0 – 3.9	NRS Level 3 4.0 – 5.9	NRS Level 4 6.0 – 8.9
1	Number and Operations: Base Ten	0.0 – 1.9	2.0 – 3.9	4.0 – 5.9	
2	Operations and Algebraic Thinking	0.0 – 1.9	2.0 – 3.9	4.0 – 5.9	
3	Measurement and Data	0.0 – 1.9	2.0 – 3.9	4.0 – 5.9	
4	Geometry	0.0 – 1.9	2.0 – 3.9	4.0 – 5.9	6.0 – 8.9
5	Number and Operations: Fractions		*3.0 – 3.9	4.0 – 5.9	
6	Expressions and Equations			4.0 – 5.9	6.0 – 8.9
7	The Number System			4.0 – 5.9	6.0 – 8.9
8	Ratios and Proportional Relationships			4.0 – 5.9	6.0 – 8.9
9	Statistics and Probability			4.0 – 5.9	6.0 – 8.9
10	Functions				*7.0 – 8.9

Florida Department of Education (2017). http://www.fldoe.org/academics/career-adult-edu/adult-edu/

Manipulatives in the Curriculum Frameworks

CCR.MA.ABE.3. Measurement and Data	
1.1 Represent and interpret data. a) Organize, represent, and interpret data with up to three categories. • Ask and answer questions about the total number of data points. • How many are represented in each category. • How many more or less are represented in one category than in another.	**2.1 Represent and interpret data.** a) Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. • Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. b) Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. • Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. c) Create a line plot to represent data. • Generate measurement data by using measuring tools marked with halves and fourths of a unit of measure (ruler). • Show the data by making a line plot, where the horizontal scale is marked off in units (whole numbers, halves, or fourths).
1.2 Measure lengths indirectly and by iterating (repeating) length units. a) Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end. b) Understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.	**2.2 Measure and estimate lengths in standard units.** a) Compare and describe how using standard (ruler) and nonstandard (thumb) units of measure relate to the size of the unit chosen. b) Estimate lengths using units of inches, feet, centimeters, and meters. c) Measure to determine how much longer one object is than another, using a standard length unit.
	2.3 Relate addition and subtraction to length. a) Represent whole numbers as lengths from 0 on a number line diagram. b) Represent whole number sums and differences within 100 on a number line diagram.

Florida Department of Education (2017). http://www.fldoe.org/academics/career-adult-edu/adult-edu/

Manipulatives in All Domains

Legend:

G – Geoboard

N – Number Line

2N – Double Number Line

C – Colored Counters

A – Algebra Tiles

Measurement and Data	The Number System

• Unit Length
• **G, N, C** - Representing Data
  ○ Picture Graph
  ○ Bar Graph
  ○ Line Plot
• **N** - Comparing Standard and Non-Standard Units of Measure
• **G, N** -Addition and Subtraction (i.e. integers or time) on a Number Line
• Operations Involving Time, Volume, and Masses
• Estimation
• **G** - Area and Square Units
• **G** - Area Model for Distributive Property
  ○ a(b+c) = a(b) + a(c)
• Area of Rectilinear Composite Shapes
• Perimeter
• **G** - Same Area, Different Perimeter
• **G** - Same Perimeter, Different Area
• Two- and Three-Dimensional Composite Shapes
• **N** - Conversion between quantities within the same measurement system
	• Factors and multiples
	• **G, A** - Greatest Common Factor and the Distributive Property
	• **N, C** - Integers – real world context
	• **G, N, C** - Meaning of zero – real world context
	• Rational Numbers – points on a number line; quotient of integers with non-zero divisors; decimal form (terminating or repeating)
	• Operations on rational numbers (word problems)
	• **G** - Extend number lines to coordinate axes – points on a plane (ordered pairs)
	• **G** - Quadrants of a Coordinate Plane
	• Additive Inverse
	• **G** – Reflections of ordered pairs on coordinate grid
	• **G, N** - Plotting points on a number line/coordinate plane
	• **N** - Absolute Value – distance from zero and real world applications
	• **N** - Ordering rational numbers (real-world context)

Appendix E

Math Manipulative Applications to Various ABE Topics

Legend:

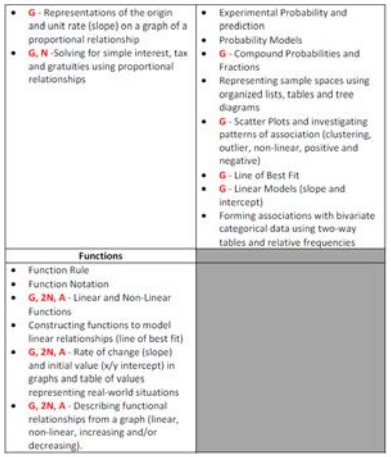
G – Geoboard

N – Number Line

2N – Double Number Line

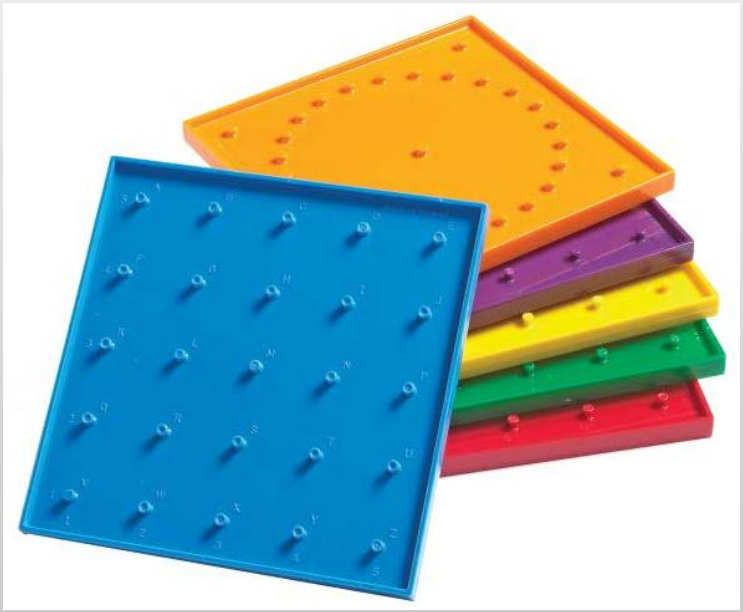
C – Colored Counters

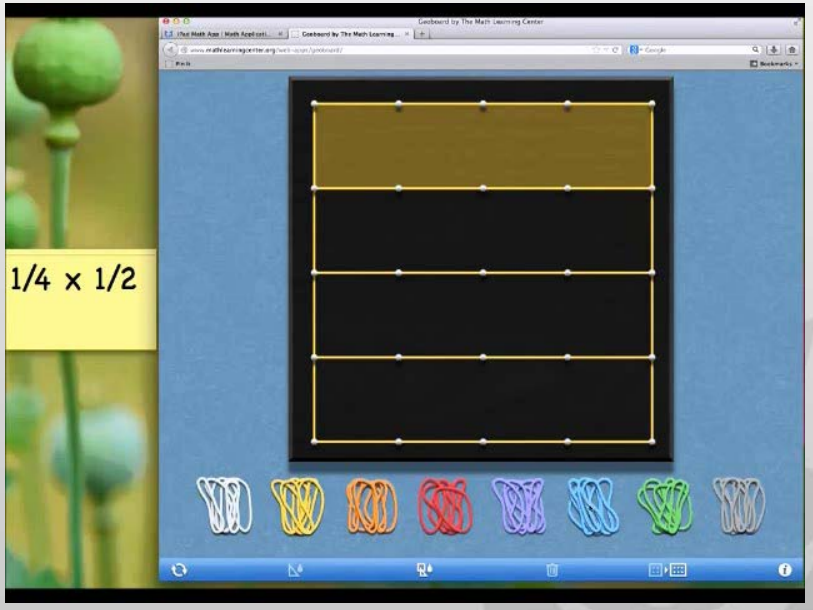
A – Algebra Tiles



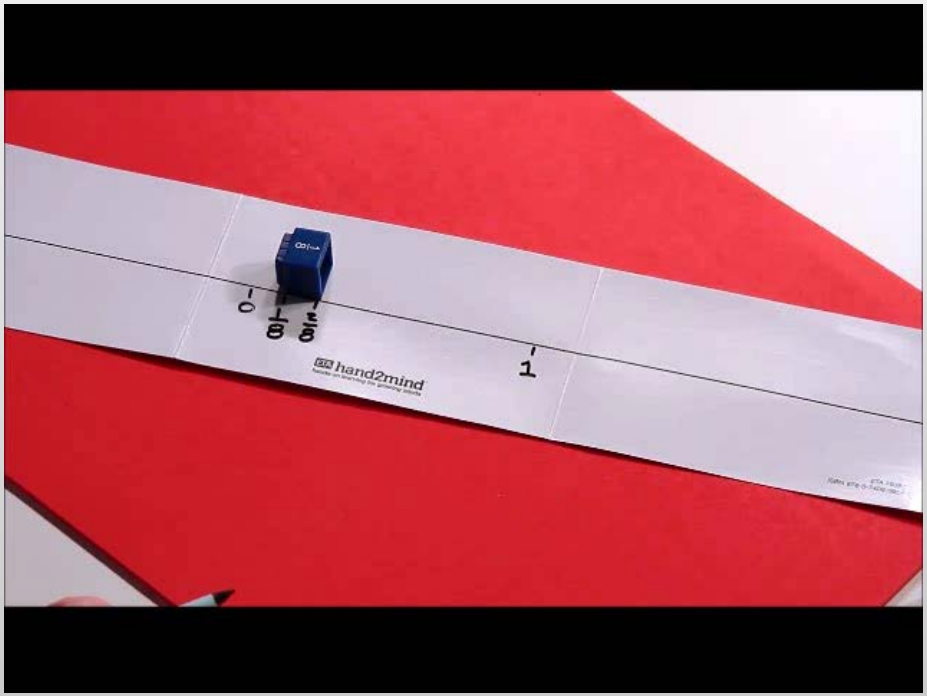
IPDAE (2016). College and Career Readiness Standards Workshop Part 2. www.floridaipdae.org – ABE – Workshops

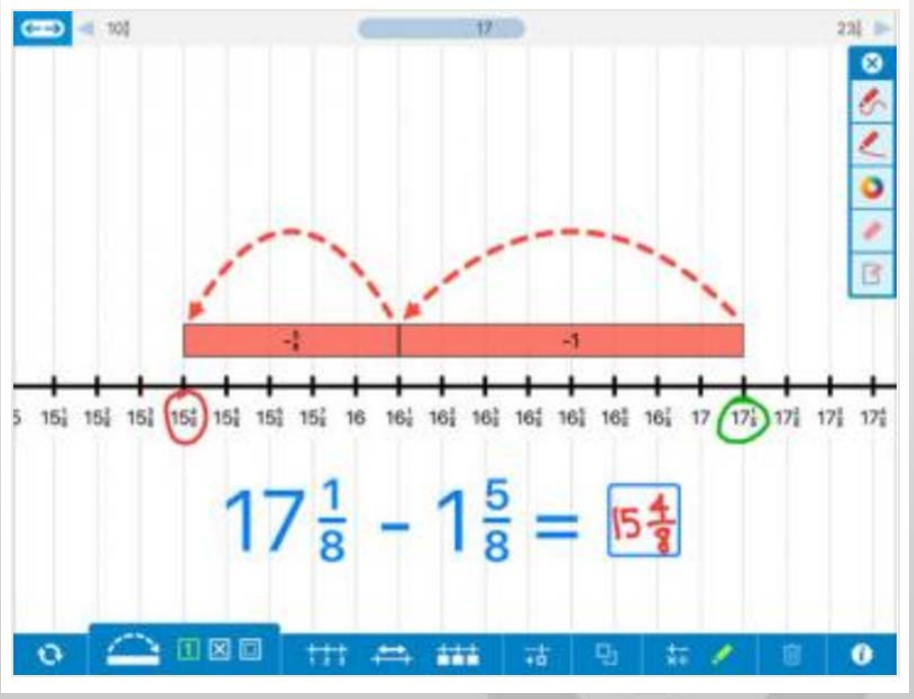
Using the Online Geoboard App to Teach Various Math Skills





Using the Number Line to Teach Various Math Skills





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Demonstration

http://nlvm.usu.edu/en/nav/vlibrary.html

ABE - Videos

Using The Algebra Tile Web App

This video will show teachers how to utilize a visual tool that can be used to model and solve single variable equations. The web app is called the Algbera Tiles Web App, made available by the National Council of Teachers of Mathematics Illuminations website (http://illuminations.nctm.org). This video was developed to supplement the resources from the ABE Mathematics CCRS face-to-face workshops. Please note that in order to fully understand this video, teachers must be familiar on the use of Algebra Tiles in modeling and solving single variable equations.

Featuring: Ronald Cruz

GED® & AHS - Videos

Manipulatives

Bonnie Goonen and Susan Pittman-Shetler

Related Documents:

Handout (PDF)

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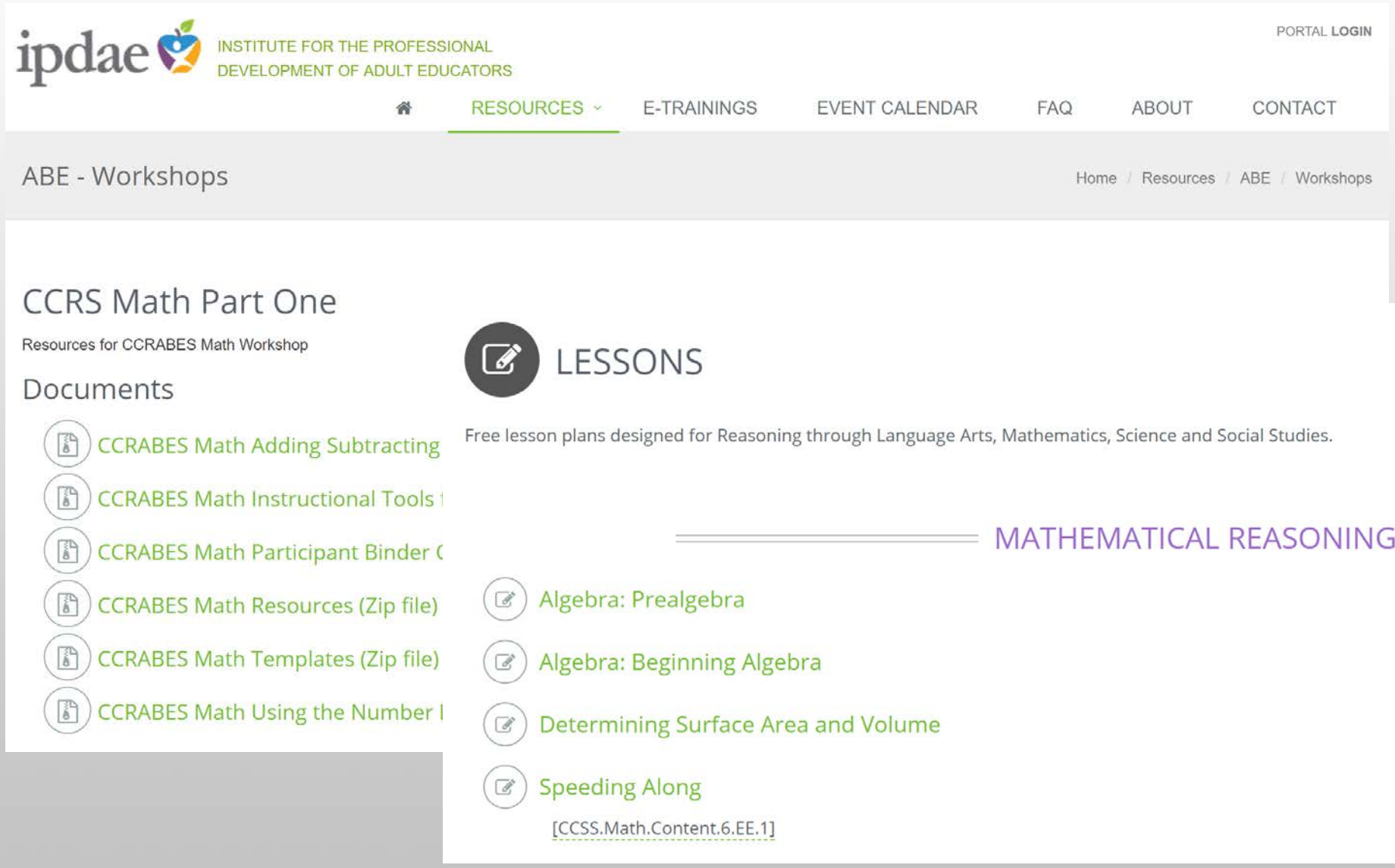
IPDAE Resources



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IPDAE Resources

CCRS Math Part One

Resources for CCRABES Math Workshop

Documents

- CCRABES Math Adding Subt
- CCRABES Math Instructiona
- CCRABES Math Participant E
- CCRABES Math Resources (Z
- CCRABES Math Templates (Z
- CCRABES Math Using the Nu

CCRS ABE Math Part 2 Workshop Documents

Resources for CCRS ABE Math Workshop.

- CCRS ABE Math Part 2 Participant Packet (PDF)
- CCRS ABE Math Part 2 Functions (Zip file)
- CCRS ABE Math Part 2 Measurement and Data (Zip file)
- CCRS ABE Math Part 2 Ratios and Proportion (Zip file)
- CCRS ABE Math Part 2 Resources (Zip file)
- CCRS ABE Math Part 2 Statistics and Function (Zip file)
- CCRS ABE Math Part 2 Templates (Zip file)

Key Ideas to Remember

- The use of manipulatives is the means and NOT the outcome for classroom instruction.
- Visual and abstract representations of math concepts must be taught side-by-side the use of manipulatives.
- Allow students to explore and experiment.
- Manipulatives could be household items or objects.
- The success of students depend on the quality of teacher-conducted instruction, not the quality or type of manipulatives.
- Lessons using manipulatives must be carefully planned.
- The way the teacher visualizes the use and representation of manipulatives is not always the same as the students.

IPDAE would like to know what you think!

https://www.surveymonkey.com/r/JLYGC89

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Thank you for your participation!