

## Unlocking the Learning to Build Resilient Learners in Math


### Pre-Workshop Overview and Support

October 16, 2019


[www.floridaipdae.org](http://www.floridaipdae.org)

This training event is supported with federal funds as appropriated to the Florida Department of Education, Division of Career and Adult Education for the provision of state leadership professional development activities.

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### Your Facilitator




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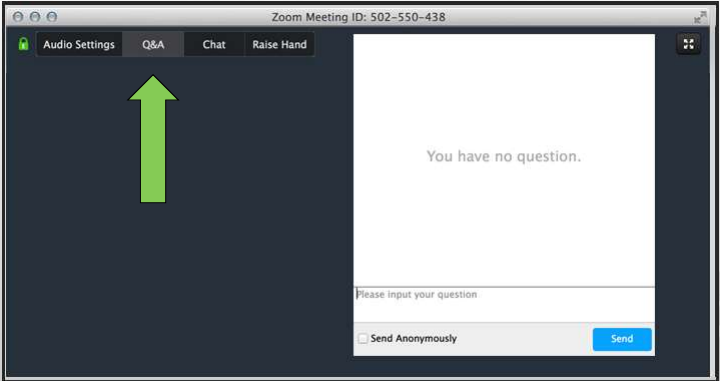
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 Webinar **Things to Remember**

- If you have a question, please type it into the **Q&A** option.




- Attendee microphones will be muted. You will be in **listen only** mode.
- Today's presentation is being **recorded**. It will be archived and available on the IPDAE website within 48 hours.



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 Webinar **Objectives**


- Use manipulatives to develop number sense and conceptual understanding of fractions.
- Model interactive and engaging strategies that enhance conceptual understanding and retention in mathematics vocabulary.



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## Agenda

- I. Why Manipulatives?
- II. What are Fraction Squares?
- III. Getting Familiar with Fraction Squares
- IV. Adding and Multiplying Fractions
- V. Simplifying Fractions
- VI. Comparing Fractions
- VII. Other Concepts Developed by Fraction Squares
- VIII. What's to Expect in Regional Workshops
- IX. IPDAE Resources



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## Manipulatives in History

### Ancient Times

Counting Board




Abacus Board



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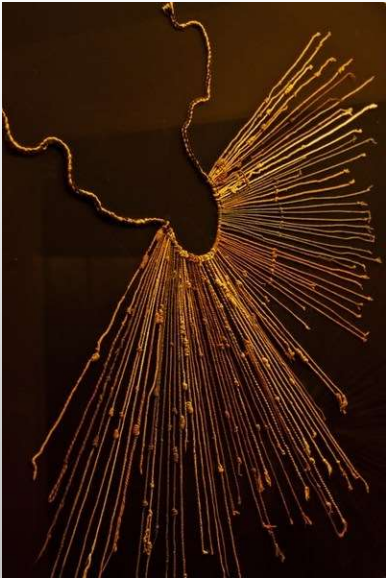


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Manipulatives in History

### Ancient Times

Quipu



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Manipulatives in History

### 1800's – 1900's



Friedrich Froebel



Maria Montessori

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Modern Day Manipulatives

### Counting Board



### Abacus






### Math Manipulatives


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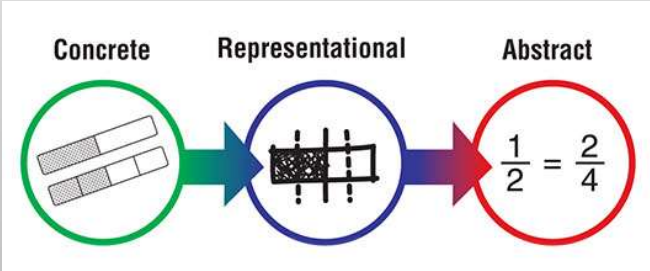
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Theory of Learning




Concrete stage	Representational stage	Abstract stage
<p>A mathematical concept is introduced with manipulatives; students explore the concept using the manipulatives in purposeful activity.</p>	<p>A mathematical concept is represented using pictures of some sort to stand for the concrete objects (the manipulatives) of the previous stage; students demonstrate how they can both visualize and communicate the concept at a pictorial level.</p>	<p>Mathematical symbols (numerals, operation signs, etc.) are used to express the concept in symbolic language; students demonstrate their understanding of the mathematical concept using the language of mathematics.</p>




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Skills Targeted by Manipulatives




- Sorting — a pre-mathematical skill that aids in comprehension of patterns and functions
- Ordering — a pre-mathematical skill that enhances number sense and other math-related abilities
- Distinguishing Patterns — the foundation for making mathematical generalizations
- Recognizing Geometric Shapes (and understanding relationships among them)
- Making Measurements (using both nonstandard and standard units with application to both two and three-dimensional objects)
- Understanding the Base-Ten System of Numbers


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Skills Targeted by Manipulatives

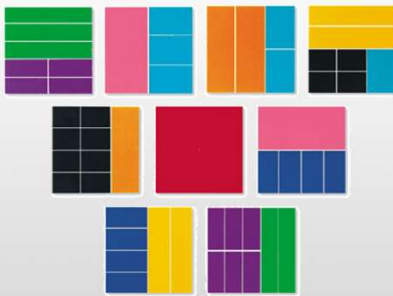



- Comprehending Mathematical Operations — addition, subtraction, multiplication, division
- Recognizing Relationships Among Mathematical Operations
- Exploring and Describing Spatial Relationships
- Identifying and Describing Different Types of Symmetry
- Developing and Utilizing Spatial Memory
- Learning About and Experimenting with Transformations
- Engaging in Problem-Solving
- Representing Mathematical Ideas in a Variety of Ways
- Connecting Different Concepts in Mathematics
- Communicating Mathematical Ideas Effectively


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## INTRODUCTION TO FRACTION SQUARES



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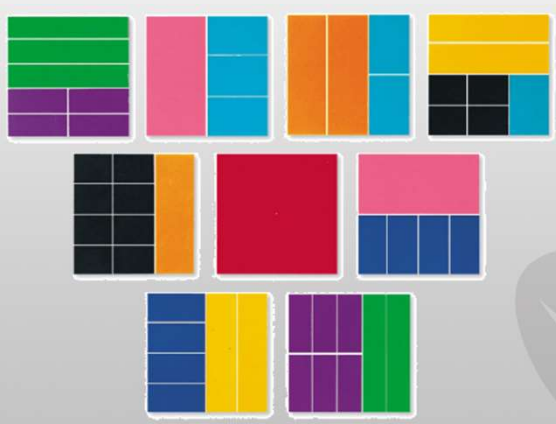
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### What are Fraction Squares?


**Fraction squares** are colored squares divided into a range of fractional pieces. They are used to illustrate how fractions combine to form a whole.



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Fraction Squares Pieces

whole




This piece is the base piece. It is used to compare combinations of other pieces to a whole.

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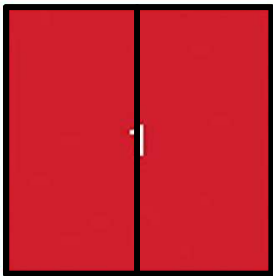
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Fraction Squares Pieces


Whole




Halves



Whole




Thirds



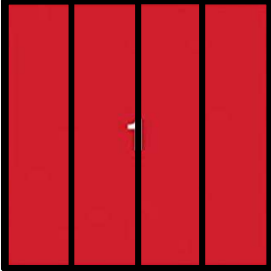
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### Fraction Squares Pieces


Whole



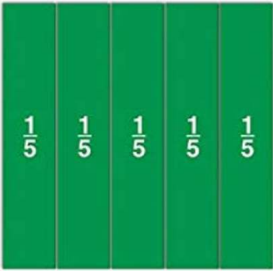
Quarters




Whole



Fifths

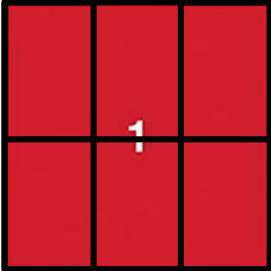


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### Fraction Squares Pieces

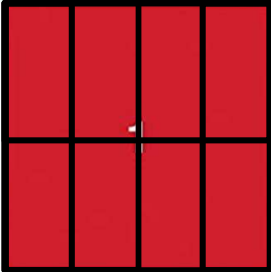
Whole




Sixths




Whole



Eighths

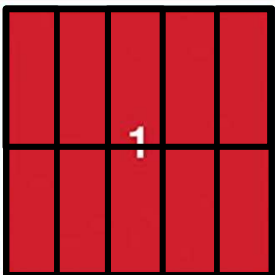


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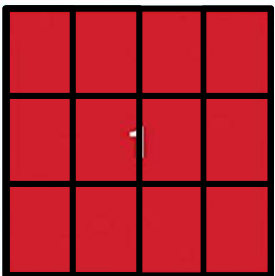


### Fraction Squares Pieces

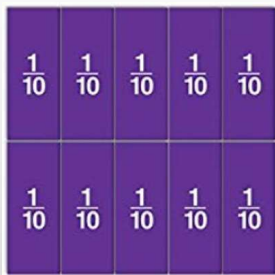
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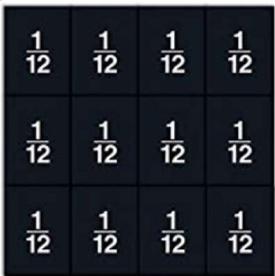
Whole




Tenths



Twelfths

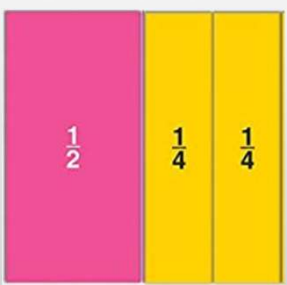


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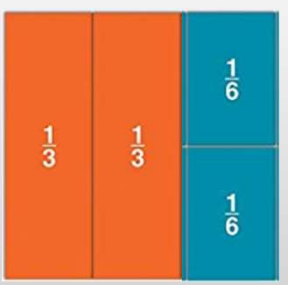
### Addition of Fractions

#### Fraction Sums Equal to a Whole




$$\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$$

$$\frac{1}{2} + \frac{2}{4} = 1$$



$$\frac{1}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6} = 1$$


$$\frac{2}{3} + \frac{2}{6} = 1$$



$$\frac{1}{2} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = 1$$

$$\frac{1}{2} + \frac{4}{8} = 1$$

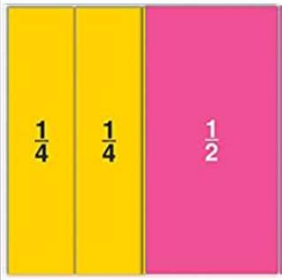
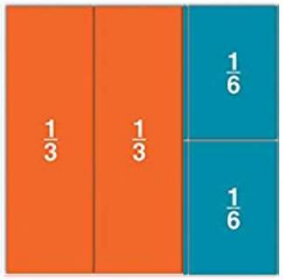

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Addition of Fractions

### Fraction Sums Equal to a Whole


Students may overlap each sum on top of a whole to ensure that the sum of fractions equals 1 whole.

*There are many more combinations of fractions that add up to 1 whole. Can you name a few?*


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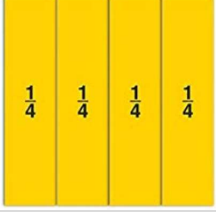
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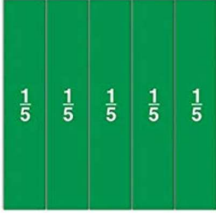
Multiplication of Fractions

### Fraction Products that Equal to a Whole

$2 \left( \frac{1}{2} \right) = 1$ 



$4 \left( \frac{1}{4} \right) = 1$ 


$3 \left( \frac{1}{3} \right) = 1$ 


$5 \left( \frac{1}{5} \right) = 1$ 


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## Multiplication of Fractions

Fraction Products that Equal to a Whole

$6\left(\frac{1}{6}\right) = 1$ 

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$


$8\left(\frac{1}{8}\right) = 1$ 

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

*Notice the pattern? Could you formulate a rule?*

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## Products & Sums of Fractions

Model the indicated operation with fraction pieces and perform the operation. Use other fraction pieces to verify answer.

$\frac{1}{4} + \frac{2}{8} + \frac{2}{8} = \frac{3}{4}$

$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$
	$\frac{1}{8}$	$\frac{1}{8}$

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
---------------	---------------	---------------

$\frac{1}{3} + \frac{2}{6} = \frac{2}{3}$

$\frac{1}{3}$	$\frac{1}{6}$
	$\frac{1}{6}$

$\frac{1}{3}$	$\frac{1}{3}$
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## Products & Sums of Fractions

Model the indicated operation with fraction pieces and perform the operation. Use other fraction pieces to verify answer.

$9\left(\frac{1}{12}\right) = \frac{3}{4}$

$6\left(\frac{1}{10}\right) = \frac{3}{5}$

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## Simplifying Fractions

Model the indicated fraction and use the fraction pieces to reduce it to simplest form.

$\frac{6}{12} = \frac{2}{4} = \frac{1}{2}$

$\frac{4}{6} = \frac{2}{3}$

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## Comparing Fractions

Use the fraction square pieces to compare the following fractions.

$\frac{2}{8} < \frac{2}{6}$

$\frac{2}{8} + \frac{2}{6} > \frac{1}{2}$

$\frac{1}{2} < \frac{2}{3}$

$\frac{1}{2} < \frac{1}{3} + \frac{1}{4}$

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## Math Skills in Using Fraction Squares

- Addition of Fractions
- Multiplication of Fractions
- Simplifying Fractions
- Comparing Fractions
- Equivalent Fractions
- Congruence
- Areas of Rectangles
- Composite Shapes (Area Addition)
- Solving (Word) Problems – when used in modeling real-world scenarios with fraction squares

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## Other Areas Improved by Manipulatives


- verbalizing mathematical thinking
- discussing mathematical ideas and concepts
- relating real-world situations to mathematical symbolism
- working collaboratively
- thinking divergently to find a variety of ways to solve problems
- expressing problems and solutions using a variety of mathematical symbols
- making presentations
- taking ownership of their learning experiences
- gaining confidence in their abilities to find solutions to mathematical problems using methods that they come up with themselves without relying on directions from the teacher

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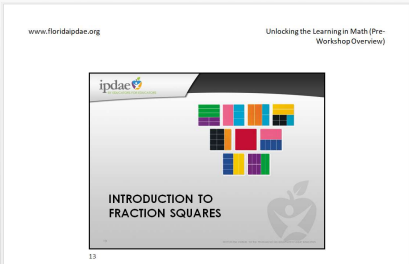
Heddens; Picciotto, 1998; Sebesta and Martin, 2004

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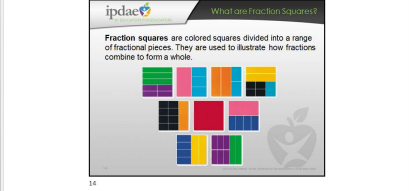
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## PPT Slides and Activity Book

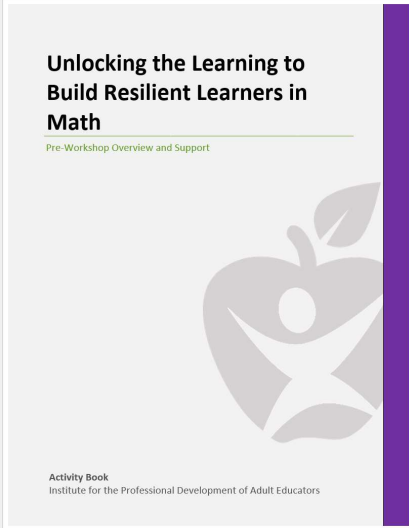


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Presented by: Ronald Cruz



Activity Book  
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Regional Workshops


Go to the IPDAE Website's Event Calendar to find out when and where regional workshops are being held that is close to your district.




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Regional Workshops


- Escambia
- Leon
- Alachua
- Seminole
- Osceola
- Pinellas
- Indian River
- Highlands
- Collier
- Miami-Dade



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## What to Expect

- Close Reading Strategies for Math
- Word Problems
- Problem-Solving Techniques
- Math Matrix
- Math Resource Activities
- Language in Mathematics
- Writing in Mathematics
- Math Vocabulary
- Math Games



### Unlocking the Learning to Build Resilient Learners through Mathematics


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## Other IPDAE Resources

Continue learning and trying out new strategies presented through:

- IPDAE's Webinar Wednesdays
- Grab and Go Videos
- Online Training Modules
- Lesson Plans
- Toolkits



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IPDAE Resources

### English for Speakers of Other Languages (ESOL) Toolkit

- Introduction
  - The Wonderful World of the Internet for ESOL Instruction
- Integrated Education
- Math
- Reading
- Research and Guides
- Speaking and Listening
- Writing
- Complete Toolkit



 <p>CCRS ELA - Module 1</p> <p>E-Learning</p>	 <p>CCRS ELA - Module 2</p> <p>E-Learning</p>	 <p>CCRS ELA - Module 3</p> <p>E-Learning</p>
 <p>CCRS Math - Module 1</p> <p>E-Learning</p>	 <p>FDOE Policies on CASAS for Florid...</p> <p>E-Learning</p>	 <p>FDOE Policies on TABE Assessme...</p> <p>E-Learning</p>


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Evaluation

**IPDAE would like to know what you think!**  
**Please complete this quick survey.**



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**Thank You**

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**Thank you for your participation!**



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