Math Grab Bag

Fall 2018
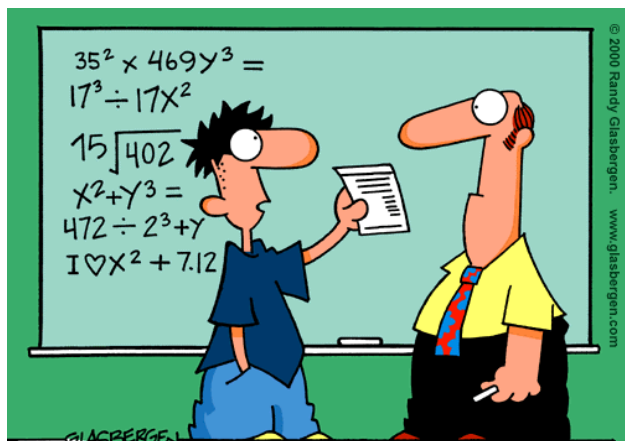
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Session Objectives

- Discuss the challenges students have with key concepts in math
- Focus on the foundational "must haves" in mathematical reasoning
- Review tips and strategies for helping students build consistency in math
- Share resources

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It Really Isn't Genetic

"I HAD MY DOCTOR DO A D.N.A. BLOOD ANALYSIS. AS I SUSPECTED, I'M MISSING THE MATH GENE."

In the Classroom, We Often…

- Make assumptions about the presence or absence of foundational skills
- Introduce new concepts too rapidly
- Insufficiently support explanations and activities
- Provide insufficient practice
- Focus on facts versus concepts
- Limit access to manipulatives
- Limit connection of skills to real-life situations

What's the Problem?

Students at Level 1

- Have limited, but developing proficiency
- Perform some math skills at a basic level, such as
  - Putting fractions/decimals on a number line
  - Solving two-step arithmetic problems
- Are inconsistent in the application of skills
- Lack fluency in basic operations and mathematical properties

GED® Test: Mathematical Reasoning Performance Level Descriptors What Your Score Means: Level 1 — Below Passing

Test-takers who score at this level typically have a limited but developing proficiency in demonstrating skills in the following categories: number sense and computation, geometric measurement, data analysis and statistics, and algebraic expressions and functions.

Test-takers at the Below Passing level typically demonstrate the following skills:

Quantitative Problem Solving with Rational Numbers

- Apply number properties involving multiples and factors at a limited and inconsistent level
- Solve real-world problems using rational numbers at a limited and inconsistent level
- Compute unit rates at a limited and inconsistent level

Quantitative Problem Solving in Measurement

- Compute the area and perimeter of triangles and rectangles at a limited and inconsistent level
- Determine side lengths of triangles and rectangles when given area or perimeter at a limited and inconsistent level
- Represent, display, and interpret categorical data in circle and bar graphs
- Represent, display, and interpret categorical data in tables and scatter plots

Algebraic Problem Solving with Expressions and Equations

- Evaluate linear expressions
- Write linear expressions to represent context at a limited and inconsistent level
- Evaluate polynomial expressions at a limited and inconsistent level
- Write rational expressions to represent context at a limited and inconsistent level
- Solve real-world problems involving linear equations at a limited and inconsistent level
- Solve algebraic and real-world problems involving systems of equations

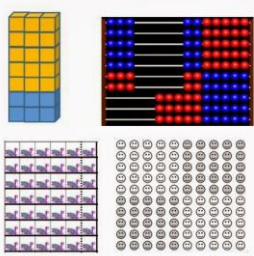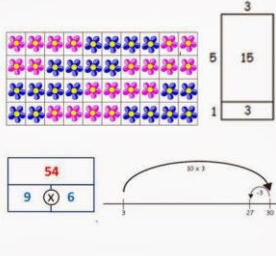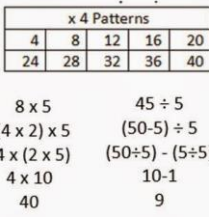**Algebraic Problem Solving with Graphs and Functions**

- Locate and plot points in the coordinate plane

https://ged.com/educators_admins/teaching/teaching_resources/plds/

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Our students need . . . Essential understanding

Concrete	Representational	Abstract
Students manipulate hands-on, concrete materials	Students draw and observe diagrams, or watch the teacher touching and moving hands-on materials	Students use numbers and mathematical symbols

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Foundational Skills: Building Number Sense

Use a Number Line

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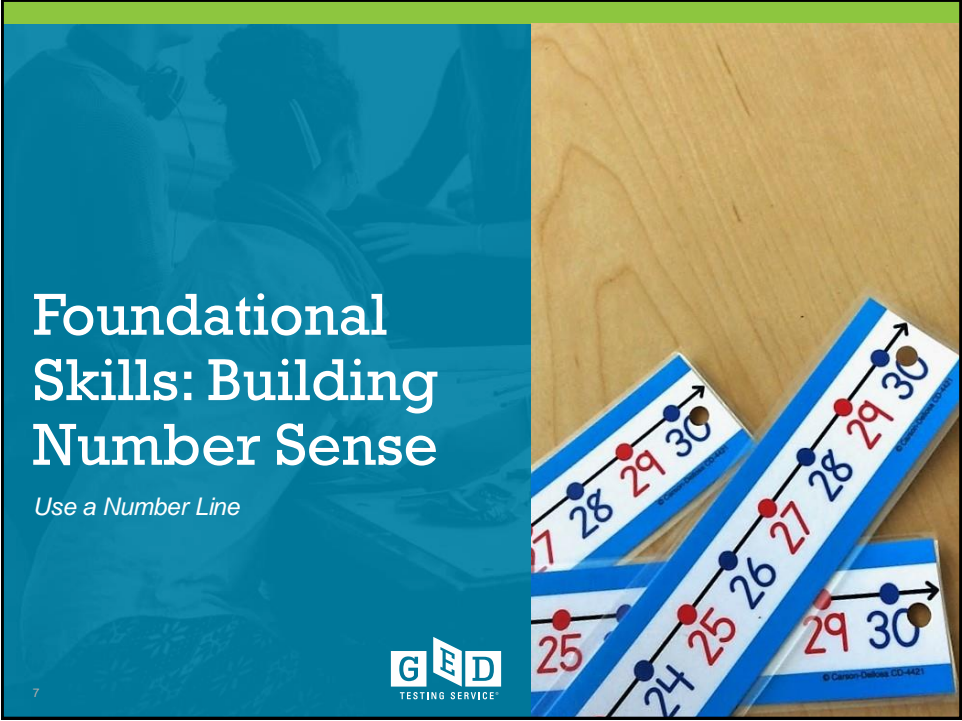
Students with Number Sense…

- Think and reason flexibly with numbers
- Use numbers to solve problems
- Spot unreasonable answers
- Understand how to put numbers together and take them apart
- Understand number relationships

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But they are just numbers…

Type of Number	Quick Description
Counting Numbers	{1, 2, 3, …}
Whole Numbers	{0, 1, 2, 3, …}
Integers	{…, -3, -2, -1, 0, 1, 2, 3, …}
Rational Numbers	p/q – p and q are integers, q is not zero
Irrational Numbers	π – 3.14159265358979323856… cannot be written as a simple fraction $\sqrt{3}, \ \sqrt{99}$
Real Numbers	Rational and Irrational

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Defining Numbers

Choose a Number Set:
☑ Counting   ☑ Integers   ☑ Rationals   ☐ Real

zoom ⊕

-3.5   -2.5   -1.5   -.5   .5   1.5   2.5   3.5
-7/2 -3 -5/2 -2 -3/2 -1 -1/2 0 1/2 1 3/2 2 5/2 3 7/2

Rational numbers are numbers that can be written as the ratio of two integers.

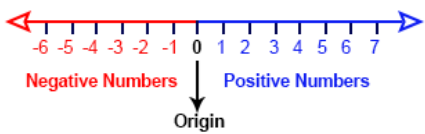Example: 5/2 (2.5), -1/4 (-.25), etc.

Open Instructions

https://unctv.pbslearningmedia.org/resource/mgbh.math.ns.numbline/building-a-number-line/#.WU1B-IWcHnM

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The Number Line

- Provides a model for basic operations for all rational numbers
- Is a spatial object
- Allows students to situate themselves spatially in mathematics
- Permits students to conceptualize mathematics

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# Just a Sample

Formula Sheet   Calculator Reference

Julia wants to spend $100 or less ordering shirts from an online company. The company charges a $5 shipping fee for any order. The inequality $5 + 15n \leq 100$ represents the number of shirts, n, Julia can order from the online company. Graph all possible numbers of shirts that Julia can buy.

Click on the number line to plot the point(s).

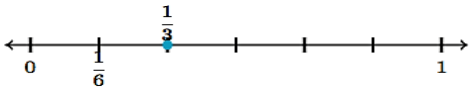(NOTE: To remove a point, place the arrow over the point and click the left mouse button.)

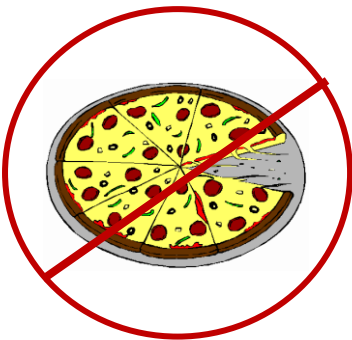1 2 3 4 5 6 7 8 9 10
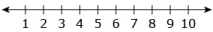
← Previous | Next →

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Fractions

Students need to learn to locate a *fraction* on a number line.

Use the number line so students understand that fractions are numbers and not just part of a pizza.
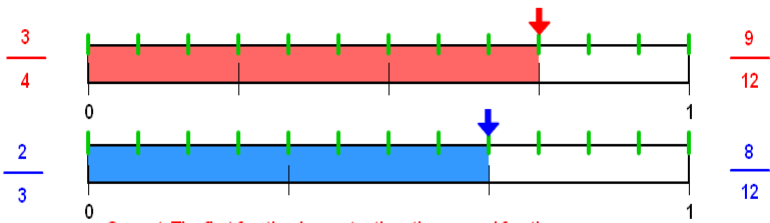
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Can Students Use a Number Line?

The fractions 3/4 and 2/3 are pictured with number lines below:

$\frac{3}{4}$ $\frac{9}{12}$

$\frac{2}{3}$ $\frac{8}{12}$

Correct. The first fraction is greater than the second fraction.

$\frac{3}{4}$ [>] $\frac{2}{3}$

"Five out of four people have trouble with fractions."
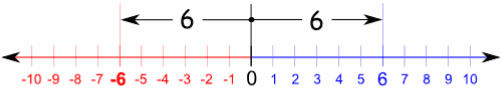-- Steven Wright

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Absolute Value

Absolute Value means how far a number is from 0.
- Remove any negative sign and think of all numbers as positive
- Recognize symbol used to represent absolute value

$$|-5| = 5$$

$$|7| = 7$$

"6" is 6 away from zero, and "−6" is **also** 6 away from zero.

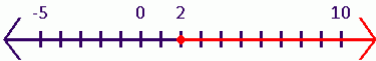So the absolute value of 6 is **6**, and the absolute value of −6 is also **6**
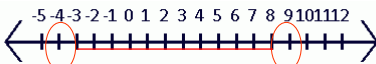
15

Inequalities

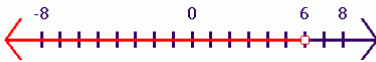An inequality is a math statement that defines a range of values.

Jeffrey runs at least two miles every day.

On November 28, the temperature in North Pole, Alaska is expected to be greater than -4° and less than 9°
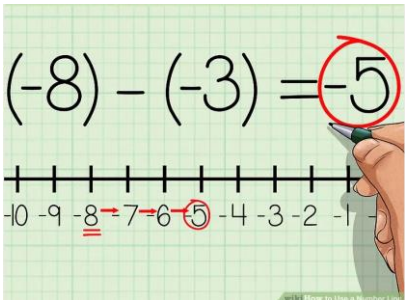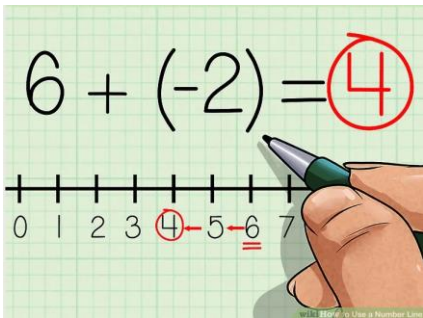
$T < 6$

https://www.gedtestingservice.com/educators/exploring-the-2014-ged-test-webinar-archive
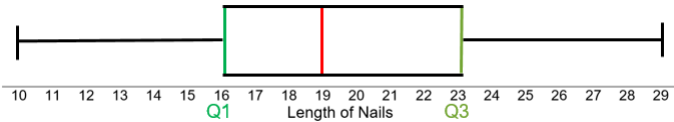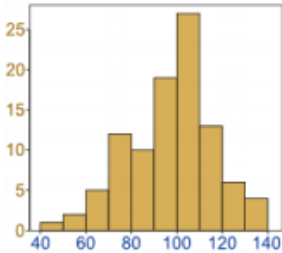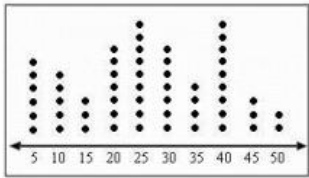
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Operations with Positive and Negative Integers

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Data Displays

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Number Lines on the TABE 11/12 They're Everywhere!

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Level B (NRS Level 2)
- 3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram
- 3.MD.1 Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
- 2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line.

Level C (NRS Level 3)
- 6.SP.4 Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
- 5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system . . .
- 6.EE.8 Write an equality . . .represent solutions of such inequalities on number line diagrams.

Level D (NRS Level 4)
- 6.NS.6 Understand a rational number as a point on the number line . . .
- 7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
- 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram . . .

Level E (NRS Level 5)
- S.ID.1 Represent data with plots on the real number line.

What Is It Worth?

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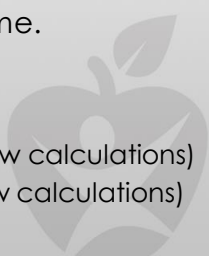
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

-13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12

- Find the value of your name by adding the values of the letters together.
  Example: Bonnie = -12 + (1) + (0) + (0) + (-5) + (-9) = -25

- Find the absolute value of your first name.
  Example: Absolute value is 25

- Write an inequality with the value of your name.
  Example: Bonnie < 0

A Little Critical Thinking Activity
- Find the highest value 5 letter word you can. (Show calculations)
- Find the lowest value 5 letter word you can. (Show calculations)

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Resources

Number Lines and the NFL – PBS
https://unctv.pbslearningmedia.org/resource/
mket-math-ns-
ratnumb/football/#.WU0tDGgrKUk

Helping with Math - Number Line Generator

http://www.helpingwithmath.com/printables/others/NumberLineGenerator01.htm

Math Warehouse – Number Line Graph Maker

http://www.mathwarehouse.com/number-lines/number-line-maker.php

Math is Fun – Number Lines (Inequalities, Operations, etc.)

http://www.mathsisfun.com/number-line.html

Annenberg Learner – Building the Number Line
http://www.learner.org/courses/learningmath/
number/session1/part_c/index.html

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IPDAE Resources

Lesson Plans
- It's All in the Game (Fractions)
- Is It the Same? (Using Equivalent Fractions)
- PreAlgebra Lessons

Grab and Go Videos
- Absolute Value
- Using the Number Line

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Workshops
- CCRS Math Workshop – Part 1

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Order Matters

The Importance of Understanding the Order of Operations

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Why Bother?

Here is your problem: 4 + 2 x 3 =

Is the answer 18 or 10?

- Avoid confusion in how problems are solved
- Set up rules of precedence or rank of operations
- Is critical to simplifying and solving different algebra problems

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Essential Understanding vs. Misconceptions

Misconception 1 - All multiplication should happen before division.

Incorrect	Correct
$12 \div 3 \times 4$	$12 \div 3 \times 4$
$12 \div 12$	4×4
1	16

GROUPINGS ( ) { } [ ]
EXPONENTS N²
MULTIPLY/**D**IVIDE ÷/×
(LEFT TO RIGHT)
SUBTRACT/**A**DD +/−
(LEFT TO RIGHT)

Misconception 2 – All addition comes before subtraction.

Incorrect	Correct
$4 + 10 - 5 + 8$	$4 + 10 - 5 + 8$
$14 - 13$	$14 - 5 + 8$
1	$9 + 8$
	17

Parenthesis

Exponents

Multiply / **D**ivide

Add + **S**ubtract

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Your Turn

What is the value of $6 \div 3 + 4 \times 2$?

So we do the division and multiplication before any addition or subtraction:

$$6 \div 3 + 4 \times 2$$
$$= 2 + 4 \times 2$$
$$= 2 + 8$$
$$= 10$$

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Your Turn

What is the value of $20 - (3 \times 2^3 - 5)$?

We start inside the **P**arentheses, and then use "**E**xponents" first:

$20 - (3 \times 2^3 - 5) = 20 - (3 \times 8 - 5)$

[Because 2^3 means $2 \times 2 \times 2 = 8$, **not** $2 \times 3 = 6$]

Next **M**ultiply:
$20 - (3 \times 8 - 5) = 20 - (24 - 5)$

Next **S**ubtract (still working inside the parentheses):
$20 - (24 - 5) = 20 - 19$

Now the **P**arentheses are completed, the last operation is **S**ubtract:
$20 - 19 = 1$

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TABE 11 and 12 Sample Question

6. Make each equation equal to 10x+11. Drag the tile into each box to construct your answer.

Expression A: $3(\boxed{} + 4) + x - 1$
Expression B: $\boxed{}(6x + 5) - 2x + 1$
Expression C: $7(\boxed{} + 2) + 3x - \boxed{}$
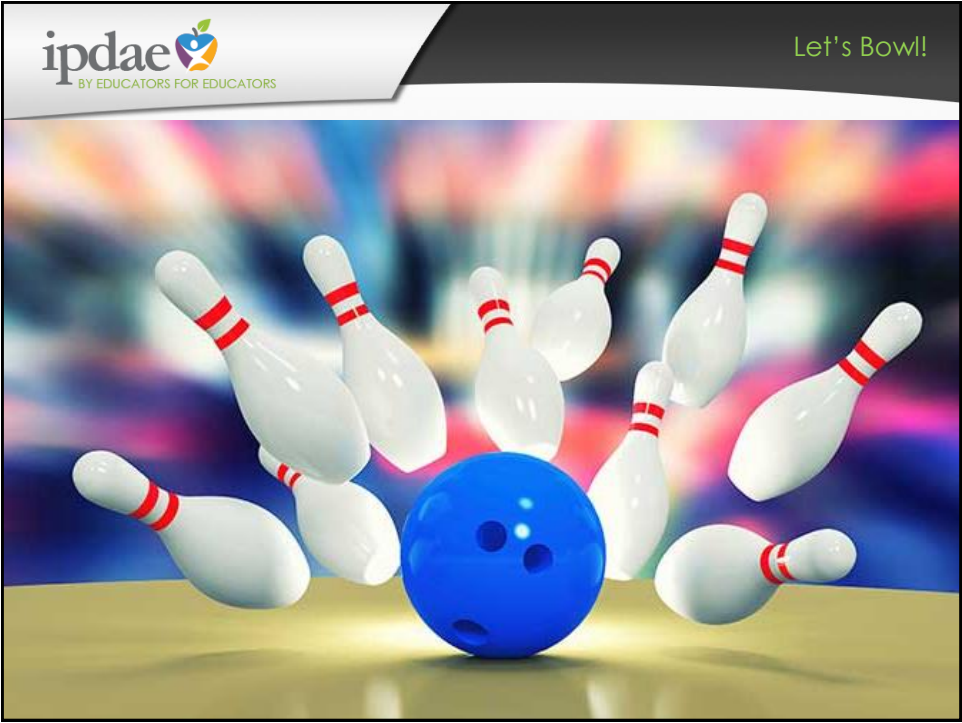
$x \quad 2x \quad 3x \quad 1 \quad 2 \quad 3$

In order to solve problems on the TABE 11 and 12, students need to use the order of operations! From the basics of numeracy to geometry to algebra, order matters!

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Let's Bowl!

IPDAE Resources

Lesson Plans
- PreAlgebra Lessons
  - Exponents and Order of Operations
- Beginning Algebra Lessons
  - Exponents, Order of Operations, Variable Expressions

Every math resource requires that students use the order of operations to solve problems.

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Do your students know the vocabulary?

Ratio – a comparison between two different values

Percent of change – ratio of the amount of change to the original amount

Percent increase – how much original amount increases

Percent decrease – how much original amount decreases

$$\text{percent change} = \frac{\text{amount of change}}{\text{original amount}}$$

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Do they understand increase vs. decrease?

- If you buy a brand new car for $15,999, drive it off the lot, and get into an accident, the car will be worth $11,499. Does the car's value increase or decrease?

- The temperature at sunrise is 71 degrees Fahrenheit . At noon, the temperature is 84 degrees Fahrenheit. At sunset, it is 69 degrees Fahrenheit. Has the temperature had an increase or decrease from sunrise to sunset?

- A scuba diver jumps off a dive boat into the water and descends 30 feet below sea level. He rises 10 feet to swim above a coral head, then swims back down 8 feet to the top of a submerged wreck. Has his depth shown an increase or decrease from his initial descent?

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Percent of Increase

- Tips
- Sales Tax
- Increase in Population

To calculate percent of increase

$$\text{percent change} = \frac{\text{amount of increase}}{\text{original amount}}$$

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Calculating a Percent of Increase

In 1981, there were 25 endangered and threatened species of reptiles in the U. S. In 2015, there were 45 species. By what percent did the number of these reptile species change from 1981 to 2016?

percent change = $\dfrac{\boxed{20}}{\boxed{25}}$ = 80%

Is the amount of change an increase or a decrease? (increase)
What is the amount of change from 1981 to 2015? (45 - 25 = 20)
What is the original amount? (25)
Divide the amount of change by the original amount (20/25 = .8)
Write the quotient as a percent (.8 = 80% increase)

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Percent of Decrease

- Discounts
- Sales
- Reduction in Population

To calculate percent of decrease

$$\text{percent change} = \frac{\text{amount of decrease}}{\text{original amount}}$$

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Calculating a Percent of Decrease

A stock was worth $18.00 a share in 2000. In 2016, the same stock was worth $7.60 a share. What was the percent of change?

58%		10.40
		18.00

Is the amount of change an increase or a decrease? (decrease)

What is the amount of change from 2000 to 2016? ($10.40)

What is the original amount? ($18.00)

Divide the amount of change by the original amount (10.40/18)

Write the quotient as a percent (.57777 . . . = 58% decrease)

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Common Errors

- Using the wrong base when calculating change
- Not being able to differentiate between a quantity change and a percentage change
- Incorrectly changing a decimal to a percent
- Confusing "percentage **of**" situations with percent increase/decrease situations
- Not reading the situation (word problem) carefully

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Resources

Art of Problem Solving: Percent Increase and Decrease Part 1

https://www.youtube.com/watch?v=vTPQV_M6tfI

Art of Problem Solving: Percent Increase and Decrease Part 2

https://www.youtube.com/watch?v=TbUlfWJ9Ohw

How to Find the Percent Change Increase: The Easy Way

https://www.youtube.com/watch?v=YWOeN7hDD3E

How To Find Percent Change Decrease: The Easy Way

https://www.youtube.com/watch?v=fwhZ8ITiReY

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Percent Increase/Decrease on the TABE 11/12

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Level D (NRS Level 4)
- 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems . . . percent increase and decrease.
- 7.EE.3 Solve multi-step real-life and mathematical problems . . . For example: If a woman making $25 an hour gets a 10% raise.

Level E (NRS Level 5)
- F.IF.8b Use properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change . . .
- F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Don't forget that standards dealing with fractions and decimals also use the skills of parts of the whole (like percents).

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Increase or Decrease?

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LET'S PLAY!

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IPDAE Resources

Lesson Plans
- It's a Bargain (Percents)
- The Consumer Price Index (Percents)
- Beginning Algebra Lessons
  - Percent and Mixture Problem Solving: Parts 1 and 2

Grab and Go Videos
- Calculating Percents

Workshops
- CCRS Math Workshop – Part 1
- Math GED Content and Strategies Training Resources

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Exponents and Roots

Oh My!

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A Continuing Problem

Students think an exponent is the same as multiplication.

I know $6^3 = 18$. Right?
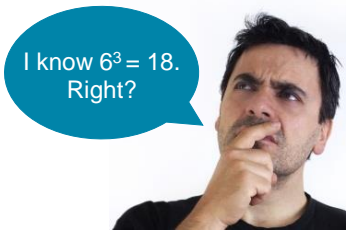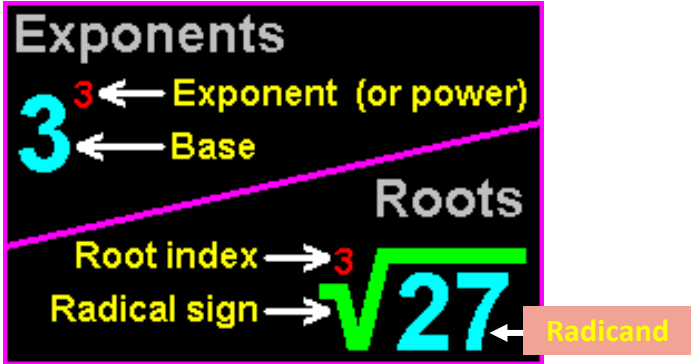
Multiplication = Repeated Addition

6 x 3 = 18
6 + 6 + 6 = 18

Exponents = Repeated Multiplication

$6^3 = 18$
6 x 6 x 6 = 216

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Do your students know the vocabulary?

Rules of Exponents Made Easier

The Math Dude – Law of Exponents - https://www.youtube.com/watch?v=g4bKGsC2IoY

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Rules of Exponents

	Rule	Example
1	$x^1 = x$	$5^1 = 5$
2	$x^0 = 1$	$5^0 = 1$
3	$x^{-1} = \dfrac{1}{x^1}$	$5^{-1} = \dfrac{1}{5}$
4	$(x^m)(x^n) = x^{m+n}$	$(x^2)(x^3) = x^{2+3} = x^5$
5	$\dfrac{x^m}{x^n} = x^{m-n}$	$\dfrac{x^3}{x^2} = x^{3-2} = x^1$
6	$(x^m)^n = x^{(m)(n)}$	$(x^3)^2 = x^{(3)(2)} = x^6$
7	$(xy)^n = x^n y^n$	$(xy)^3 = x^3 y^3$
8	$\left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$	$\left(\dfrac{x}{y}\right)^3 = \dfrac{x^3}{y^3}$
9	$x^{-n} = \dfrac{1}{x^n}$	$x^{-2} = \dfrac{1}{x^2}$

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Squares and Square Roots of Positive Rational Numbers

Recommendations for Test-takers
- Memorize the first 12 perfect squares (1, 4, 9, …, 144)
- Understand inverse relationships between pairs of squares and square roots ($12^2 = \sqrt{144}$ and $\sqrt{144} = 12$)
- Understand difference in squaring a negative number and the negative of a square number, i.e., $(-3)^2 = 9$ and $-(-3)^2 = -9$
- Practice computing with square and square roots that include fractions and decimals

MATH is RADICAL

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Simplifying Radical Expressions

- $\sqrt{9}$

- Find the prime factors $= \sqrt{3 \cdot 3}$

- Bring any pairs outside the radical = 3

$$\sqrt{9xy^2}$$
$$\sqrt{3 \cdot 3 \cdot x \cdot y \cdot y}$$
$$3y\sqrt{x}$$

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Why do students need to know this?

Mathematical Reasoning - Candidate Name   Question 5 of 16

☑ Answer Explanation Æ Symbol   Flag for Review

🖹 Formula Sheet

Type your answer in the box. You may use numbers, symbols, and/or text in your answer.

An expression is shown.

$$\sqrt{15} \cdot \sqrt{12}$$

Simplify the expression completely. Leave your answer in radical form.

(NOTE: Click the symbol selector when you need to enter the radical sign.)

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Exponents, Roots, Radicals on the TABE 11/12

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Level D (NRS Level 4)

8.EE.1 Know and apply the properties of integer (a number with no fractional part) exponents to generate equivalent numerical expressions.

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots.

8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions . . .

Level E (NRS Level 5)

A.SSE.2 Use the structure of an expression to identify ways to rewrite it.

N.RN Rewrite expressions involving radicals and rational exponents using properties of exponents.

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IPDAE Resources

Lesson Plans
- PreAlgebra Lessons
  - Exponents and Order of Operations
  - Integers
- Beginning Algebra Lessons
  - Exponents, Order of Operations, Variable Expressions

Grab and Go Videos
- Exponents

Workshops
- Building Capacity for Florida GED® Programs: Reasoning through Language Arts and Mathematical Reasoning
- CCRS Math Workshop – Part 1
- Math GED® Content and Strategies Training Resources

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Last, but not least

The Incredible Zero

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The Incredible Zero

- It is unique in representing nothingness.
- As a placeholder it gives our number system its power.
- It acquires different meaning based on its location. Think 30 versus 3,000.

The Origin of the Number Zero

http://www.smithsonianmag.com/history/origin-number-zero-180953392/#qagAYijydW3RXhhk.99

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Properties of Zero

Property	Example
a + 0 = a	4 + 0 = 4
a − 0 = a	4 − 0 = 4
a × 0 = 0	6 × 0 = 0
0 / a = 0	0/3 = 0
a / 0 = undefined (dividing by zero is undefined)	7/0 = undefined
$0^a = 0$ (a is positive)	$0^4 = 0$

http://www.mathsisfun.com/numbers/zero.html

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The Problem with Zero

$$\frac{a}{0} \qquad \frac{7}{0}$$

You can express a fraction with 0 in the denominator, but it has no meaning.

Division by zero is undefined. Mathematicians have never defined the meaning because there is no good definition.

How many times can you throw nothing into no baskets?

As many times as you want. It's just not a real number.

To learn more:
https://www.youtube.com/watch?v=NKmGVE85GUU

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Calculator-Prohibited Indicators

Do the skills look familiar?

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GED® test Calculator-Prohibited Indicators

Order Fractions and Decimals	Place the following numbers in order from greatest to least: 0.2, -1/2, 0.6, 1/3, 1, 0, 1/6
Factors and Multiples	Find the LCM that is necessary to perform the indicated operation. 7/6 – 1/4 =
Rules of Exponents	Simplify the following: $(x^3)^5$
Distance on a Number Line	Find the distance between two points -9 and -3 on a number line

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GED® test Calculator-Prohibited Indicators

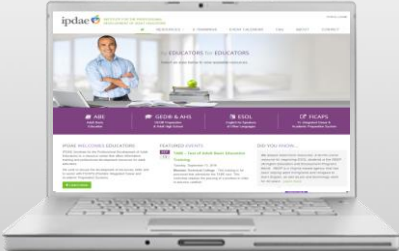| Operations on Rational Numbers	Solve: 3 (½) ÷ 3 ½ =
Squares and Square Roots of Positive Rational Numbers	Find $\sqrt{9}$ Find $\sqrt{24}$
Cubes and Cube Roots of Rational Numbers	Find $(-4)^3$
Undefined Value Over the Set of Real Numbers	Solve $(2x - 3)(x + 2) = 0$

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Webinars
- Calculator Prohibited Indicators Parts 1 and 2 (2016)

Workshops
- Building Capacity for Florida GED® Programs: Reasoning through Language Arts and Mathematical Reasoning

floridaipdae.org

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Khan Academy Math Practice Sets Aligned to TABE® 11/12 (CrowdED Learning)

- Correlates Khan Academy activity sets (over 2,000) to TABE® 11/12 test blueprints
- Will be incorporated into *SkillBlox* learning plan generation tool
- Will help instructors develop personalized learning plans

https://www.crowdedlearning.org/learn/projects

Tips for Building Foundational Skills

- Help students build their number sense
- Include opportunities for students to work together
- Provide plenty of practice with real-life situations included
- Set high expectations

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Thank you!

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