

Session Objectives



- Determine the big ideas of algebra
- Discuss the importance of teaching the basics of inequalities and functions (two High Impact Indicators)
- Connect inequalities and functions to real-world situations
- Share resources

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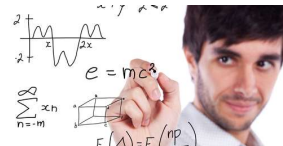
The Magic of Algebra (What's the reason?)

Think of any number.

- Multiply it by 2.
- Add 4.
- Multiply by 3.
- Divide by 6.
- Subtract the number with which you started.

You got 2!

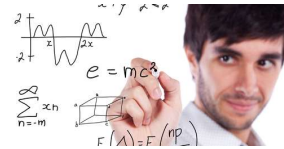
Explain with algebra why this works.



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The answer is ...



Start with the expression that describes the operations to be performed on your chosen number, x :

$$\frac{(2x + 4) \cdot 3 - x}{6}$$

and simplify the expression. You'll end up with 2, regardless of the value of x .

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Why Use Magic Tricks or Puzzles?

- They are
 - Fun
 - Non-threatening
 - Motivational
 - Engaging
- Students begin to use algebraic thinking without knowing that is what they are doing.

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Some Big Ideas in Algebra

- Variable
- Symbolic Notation
- Equality
- Ratio and Proportion
- Pattern Generalization
- Equations and Inequalities
- Multiple Representations of Functions

I'm still waiting for the
day that I will actually use

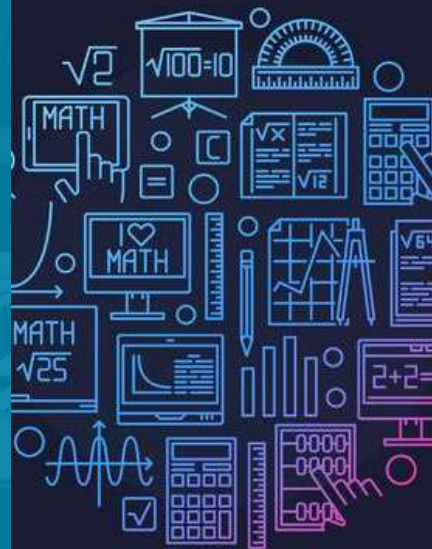


$xy + (4 - 20) >$
 $x - 5y[2 + 9 - 7]$
in real life



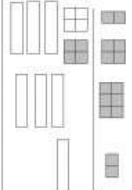
Holding Algebra in Your Hands

*Starting with the concrete by using algebra
tiles*



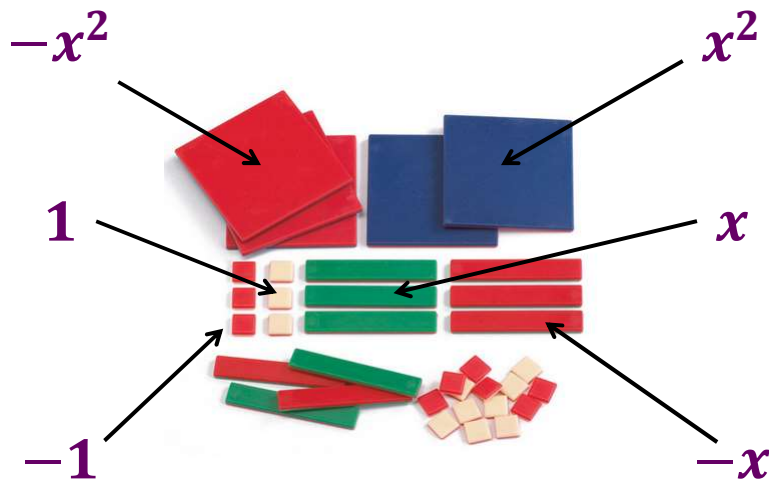
When teaching algebraic concepts, model using multiple representations

- Start with the concrete
- Represent problems using symbols, expressions, tables, and graphs
- Model real-world situations
- Complete problems different ways (flexibility in problem solving)

Equation	Tile Model	Written Description	Mathematical Procedure
$3x + 4 = -2$		<ol style="list-style-type: none"> 1. Given 2. Add 4 negatives to each side 3. Collect like terms 4. Divide each side into three equal groups 5. Simplify 6. Check 	$ \begin{array}{rcl} 3x + 4 & = & -2 \\ -4 & = & -4 \\ \hline 3x & = & -6 \\ \div 3 & = & \div 3 \\ \hline x & = & -2 \end{array} $

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Introduction to Algebra Tiles

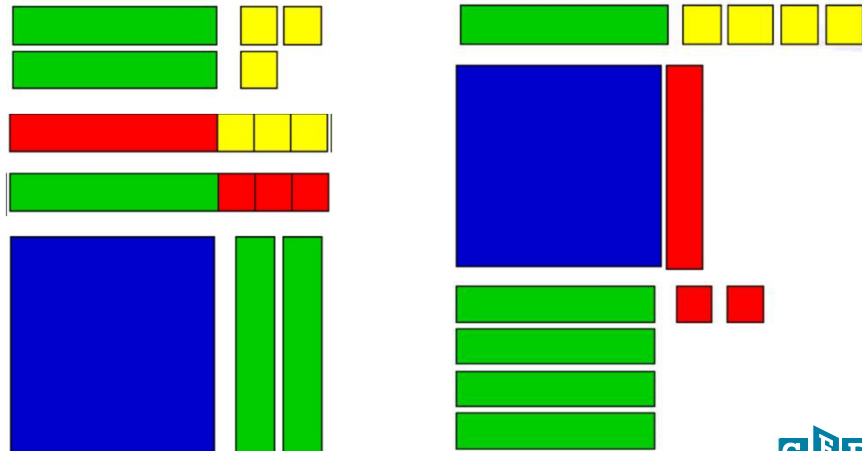


Remember, they could be called x, y, b, t, etc.



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What's My Polynomial?



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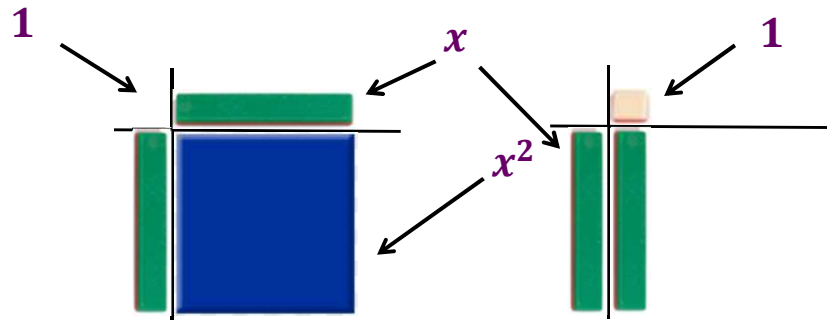
Big Ideas Using Algebra Tiles

- Adding and Subtracting Integers; Zero Principle
- Modeling Linear Expressions
- Solving Linear Equations
- Simplifying Polynomials
- Solving Equations for Unknown Variable
- Multiplying and Dividing Polynomials
- Factoring Trinomials
- Completing the Square
- Investigations

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Look at the Relationships among the Tiles



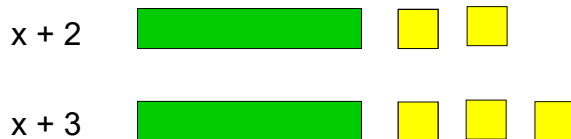
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Let's Use Algebra Tiles to Multiply Polynomials

Problem: $(x + 2)(x + 3) = ?$

Step 1: Gather all the tiles you need for each term.



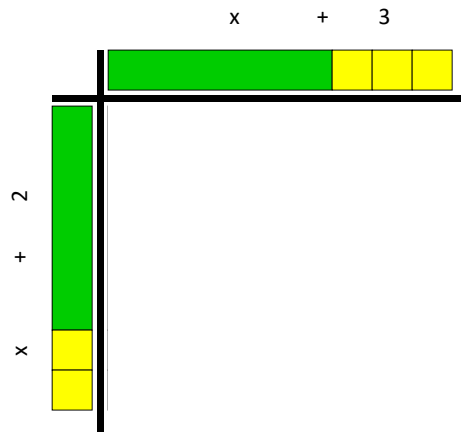
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Multiplying Polynomials

$$(x + 2)(x + 3) = ?$$

Step 2: Lay out the tiles on opposite axes of the algebra tile mat as shown below.



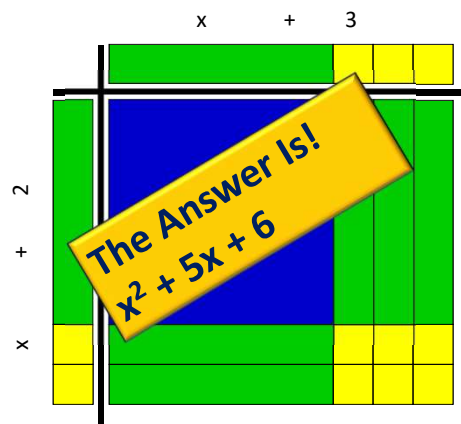
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Multiplying Polynomials

$$(x + 2)(x + 3) = ?$$

Step 3: Using other tiles, form a perfect rectangle using the tiles on axes of the product mat as the indicators for the dimensions of the rectangle as shown.

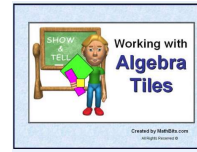


Notice that no other combination of tiles would make a perfect rectangle matching the dimension of tiles on the axes of the product mat.

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Resources



- Working With Algebra Tiles – MathBits
<http://mathbits.com/MathBits/AlgebraTiles/AlgebraTiles.htm>
- Factoring Polynomials Using Algebra Tiles - Del Mar College
<http://dmc122011.delmar.edu/math/MLC/QEPMathSeminars/FactoringTrinomialsAlgebraTilesStudentActivity.pdf>
- Multiplying Polynomials Using Algebra Tiles – Virginia Dept. of Education
http://www.doe.virginia.gov/testing/solsearch/sol/math/A/mess_a-2b_2.pdf

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Algebra Tile Apps

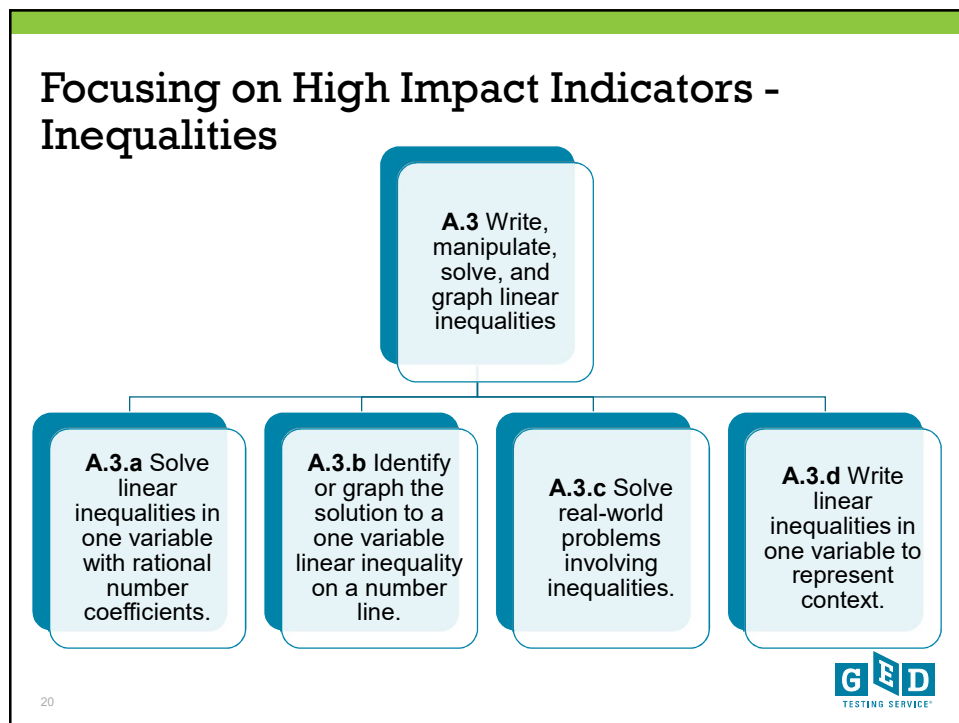
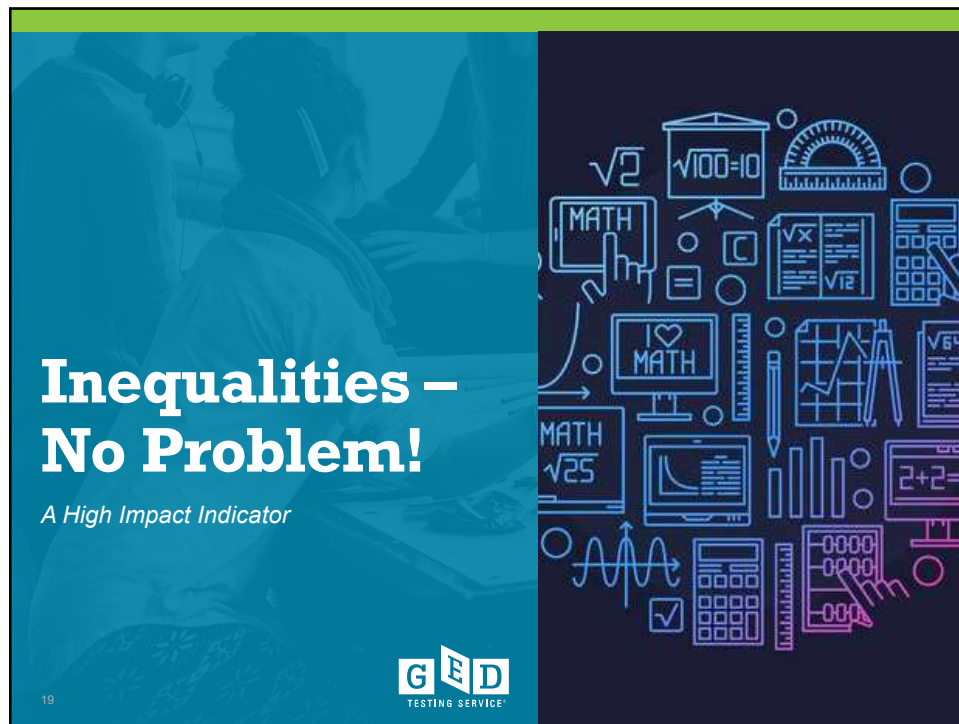


Algebra Tiles
Brainiacs, LLC
★★★★☆ 4.7, 55 Ratings
\$1.99

Don't forget phone apps!

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Can your students . . .

Solve inequalities in one variable, using the standard algorithms?

Solve a one-variable inequality and identify or create a graph on the number line of the solution?

Analyze the relationship between quantities in a real-world problem, and then create an inequality to model the problem situation?

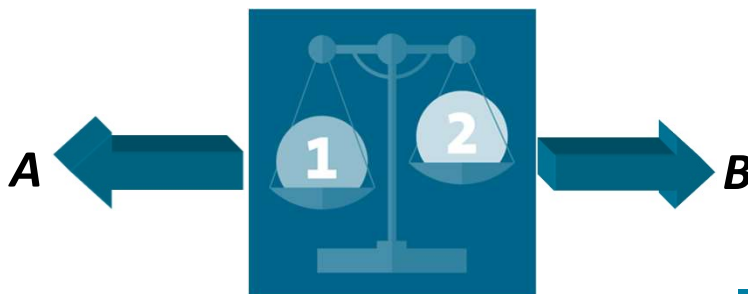
Analyze the relationship between quantities in a real-world problem, and then solve the problem through algebraic reasoning?

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What is an inequality?

An inequality is a mathematical sentence that uses symbols such as $<$, \leq , $>$, or \geq to compare two quantities.



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Inequalities Are Everywhere

Situation	Mathematical Inequality
Speed limit	Legal speed on the highway ≤ 65 miles per hour
Credit card	Monthly payment $\geq 10\%$ of your balance in that billing cycle
Text messaging	Allowable number of text messages per month ≤ 250
Travel time	Time needed to drive from home to school/work ≥ 18 minutes

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Recognize the Symbols and the Vocabulary

Term	Inequality
Coefficient	$4a > 8$
Boundary Point	A solution that makes the inequality true
Solution Set	The range of values that make the inequality true
Inclusive	$a \leq 6$
Exclusive	$a < 6$

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Rules for Solving Inequalities

1. Make the same changes to both sides of the inequality
2. Isolate the variable
3. Combine like terms
4. Use the inverse operation to remove clutter from the variable
5. If your inverse operation is multiplication or division by a negative number, reverse the inequality sign

< becomes >
> becomes <
 \leq becomes \geq
 \geq becomes \leq

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Properties of Inequalities

Addition and Subtraction

If $a > b$, then $a + c > b + c$

If $a > b$, then $a - c > b - c$



Real-life situation

Becky is older than Janet: $b > j$

Add 10 years: $b + 10 > j + 10$

Subtract 10 years: $b - 10 > j - 10$

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Properties of Inequalities

Multiplication and Division

If $a > b$, then $ac > bc$, if $c > 0$

If $a < b$, then $ac < bc$, if $c < 0$



Real-life situation

Becky is older than Janet: $b > j$

When they are twice their current age:

$$b(2) > j(2)$$

When they were half the age they are now:

$$\frac{b}{2} > \frac{j}{2}$$

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But...there is one exception

$$-3n > 12$$

If you divide or
multiply by a
negative number

$$\frac{-3n}{-3} > \frac{12}{-3}$$

reverse the
inequality symbol

$$n < -4$$

Solution: all numbers
less than -4

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Testing the Property

$$3 > 2$$

Multiply by -1

$$(-1)(3) > 2(-1)$$

$$-3 > -2 \quad \text{FALSE}$$

$$-3 < -2 \quad \text{TRUE}$$

Multiplying by a negative flipped the inequality sign from "greater than" to "less than."

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Solve the Inequality

$$4 + x < 12$$

$$4 + \boxed{x} < 12 \text{ (draw wall down inequality)}$$

$$4 + x < 12 \text{ (box in variable)}$$

$$4 + x < 12 \text{ (minus 4 both sides)}$$

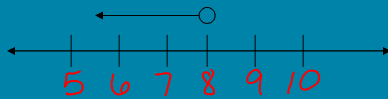
$$\begin{array}{c|c} -4 & -4 \\ \hline & x < 8 \end{array}$$

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Graph the Solution

$$x < 8$$

1. Draw a number line. Just need a few numbers on either side of the solution number.



2. Decide if open circle or closed circle. Place it above the solution number.
3. Determine which way your arrow goes by substituting a number in for the variable to make the statement true. Then draw the arrow pointing in that direction.

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Sample Question from GEDTS

Mathematical Reasoning - Candidate Name Question 13 of 16

☒ Answer Explanation ☐ Calculator ☐ Flag for Review

[Formula Sheet](#) [Calculator Reference](#)

Julia wants to spend \$100 or less ordering shirts from an online company. The company charges a \$5 shipping fee for any order. The inequality $5 + 15n \leq 100$ represents the number of shirts, n , Julia can order from the online company. Graph all possible numbers of shirts that Julia can buy.

Click on the number line to plot the point(s).

(NOTE: To remove a point, place the arrow over the point and click the left mouse button.)

[< Previous](#) [Next >](#)

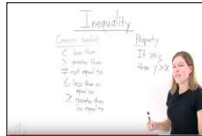
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Resources



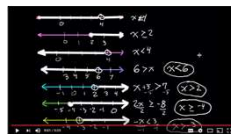
One-Variable Inequalities – Khan Academy

<https://www.khanacademy.org/math/algebra/a/one-variable-linear-inequalities>



Virtual Nerds: What is an Inequality?

<https://www.youtube.com/watch?v=wcBwdz-ZBaM>



Very Basics of Graphing Inequalities (on a number line)

<https://www.youtube.com/watch?v=nif2PKA9bXA>

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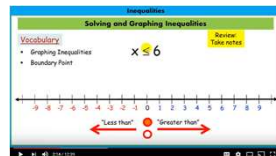


Resources



Math is Fun – Solving Inequalities

<http://www.mathsisfun.com/algebra/inequality-solving.html>



Solving and Graphing Inequalities (Excellent!)

<https://www.youtube.com/watch?v=EE2qWlyjKD0>

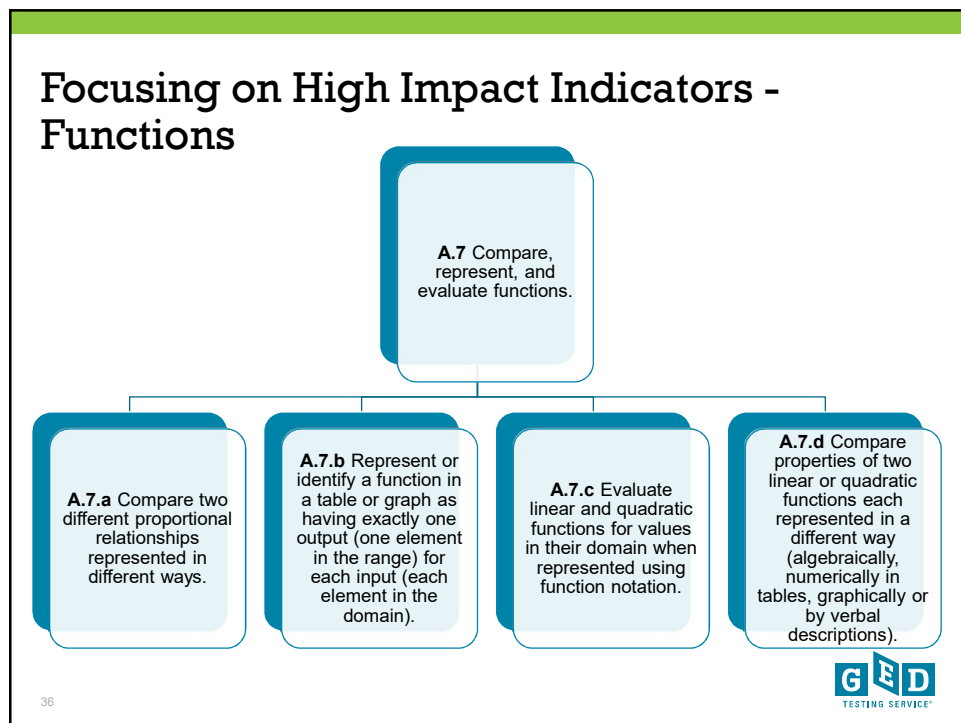


Math Dude Unit 1-4 – Solving Inequalities

https://www.youtube.com/watch?v=8hhewFQ_K0w

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Can your students . . .

Identify functions and non-functions displayed in graphs and tables, and create functions (graphs/tables)?

Substitute values for variables in functions and evaluate the resulting numerical expressions?

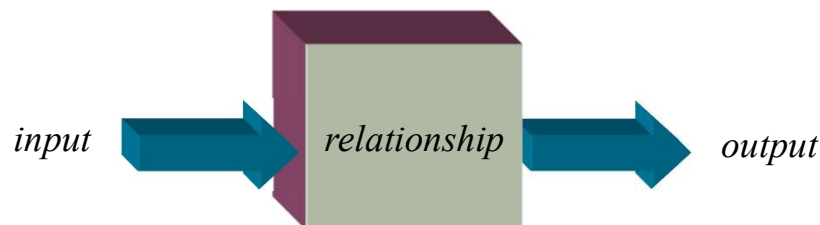
Convert functional representations from one form to another, and compare properties of the functions?

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What is a function?

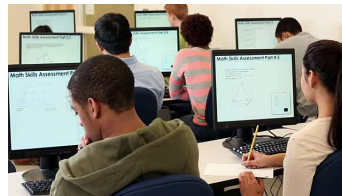
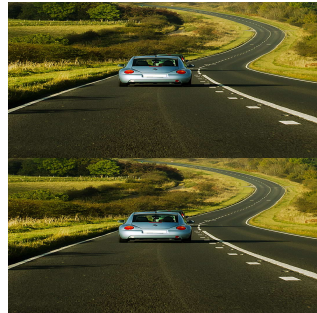
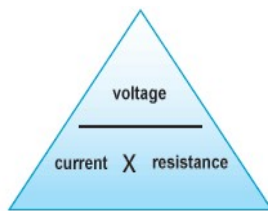
A **function** is a special relationship between a set of inputs and a set of permissible outputs. It is a useful mathematical tool.



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Functions in Daily Life

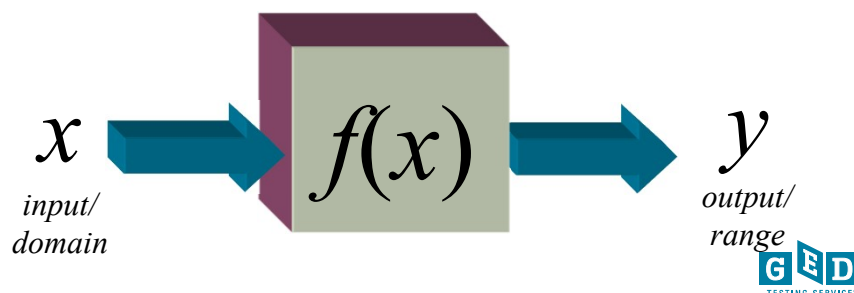


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What is a function? Looking Closer

A **function** is a relation in which each element of the domain is paired with exactly one element of the range. Another way of saying it is that there is one and only one output (y) with each input (x).



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In order for a relationship to be a function . . .

EVERY INPUT MUST HAVE AN OUTPUT

TWO DIFFERENT INPUTS CAN HAVE THE SAME OUTPUT

ONE INPUT CAN HAVE ONLY ONE OUTPUT

INPUT (DOMAIN) → FUNCTION MACHINE → OUTPUT (RANGE)

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GED TESTING SERVICE

Is it a function?

<u>Input</u>	<u>Output</u>
-3	3
1	1
3	-2
4	-2

Domain (input) = $\{-3, 1, 3, 4\}$
Range (output) = $\{3, 1, -2\}$

Function?
Yes: each input is mapped onto exactly one output

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GED TESTING SERVICE

Real World – Is It a Function?

People and Social Security Numbers

Domain	All people with a valid social security number
Range	All valid social security numbers
Is it a function?	Yes

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Real World – Is It a Function?

People and Phone Numbers

Domain	All people who have a phone
Range	Phone numbers of all people who have a phone
Is it a function?	No

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Show Functions Four Ways

x	y
-4	-11
-2	-7
0	-3
2	1
4	5

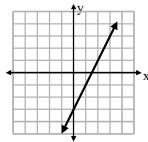
Table

$$y = 2x - 3$$

Equation

y is 3 less than twice a number x

Written Description



Graph

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Vocabulary

x	y
Input	Output
Domain	Range
Independent Variable	Dependent Variable
x	$f(x)$

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Is it a function? Creating Input/Output Tables

$\{(4,3), (-2, 10), (4, -6), (10,7)\}$

There are two inputs that are the same number, but each has a different output. A relationship does not exist, so no, it is not a function.

Input x	Output y
4	3
-2	10
4	-6
10	7

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Is it a function?

Which of the following relations are functions?

$R = \{(9,10), (-5, -2), (2, -1), (3, -9)\}$

$S = \{(6, a), (8, f), (6, b), (-2, p)\}$

$T = \{(z, 7), (y, -5), (r, 7), (z, 0), (k, 0)\}$

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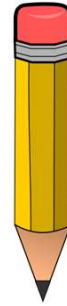


Is it a function?

Vertical Line Test: a relation is a *function* if a vertical line drawn through its graph, passes through only one point.

AKA: “**The Pencil Test**”

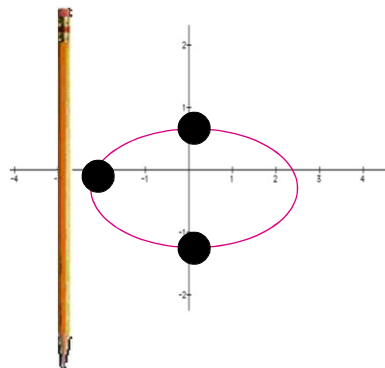
Take a pencil and move it from **left to right** ($-x$ to x); if it crosses more than one point, it is not a function.



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Vertical Line Test



Would this graph
be a function?

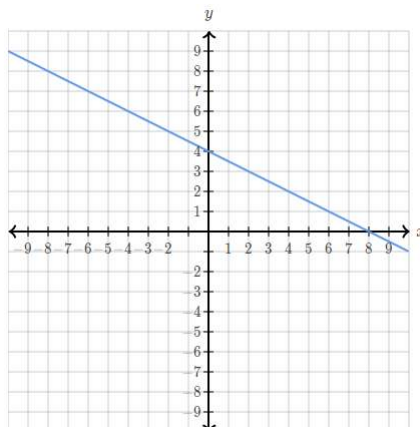
NO

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Is it a function?

Yes

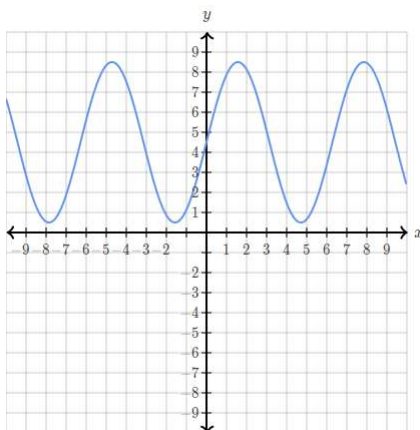


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Is it a function?

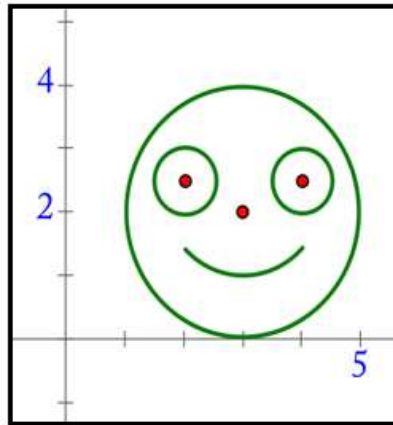
Yes



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Is it a function?



No

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Back to Real-World Situations

Is it a function?

- The relation of distance and time during a trip.
- The relation of a month to the length of daylight.
- The relation of a person's shoe size to their height.
- The relation of amount of money earned and hours worked.

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Function Notation

$$y = 3x + 2$$

$$f(x) = 3x + 2$$

Name of
Function

Input

Output

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Evaluate the Function

Find $f(-2)$.

$$f(x) = 2x^2 - 3x + 6$$

$$f(-2) = 2(-2)^2 - 3(-2) + 6$$

$$f(-2) = 2(4) - 3(-2) + 6 = 8 + 6 + 6 = 20$$

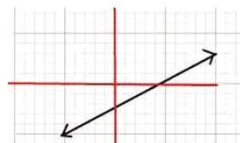
To find $f(-2)$ you need to substitute a -2 for every x value. Then carefully simplify using the order of operations.

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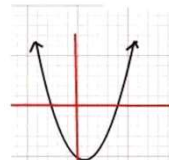
Types of Functions – The Next Step

Linear

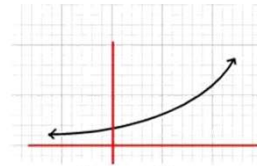


Linear

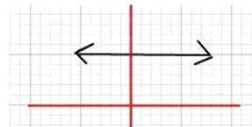
Non-Linear



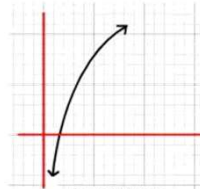
Quadratic



Exponential



Constant



Logarithmic

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A Real-World Linear Function

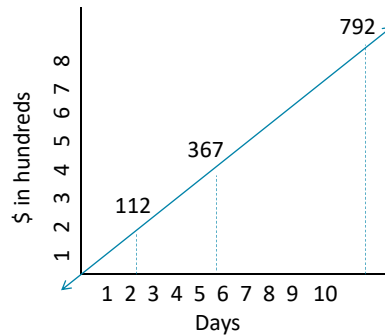
A lawyer charges a base (one time) fee of \$200 and \$75 each hour for consulting with her. Calculate the total cost of the lawyer if you consulted with her for one, two, three, four, or five hours.

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Remember Harley?

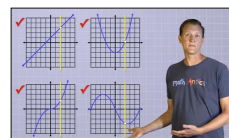
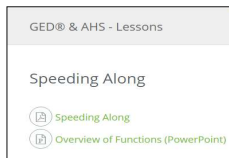
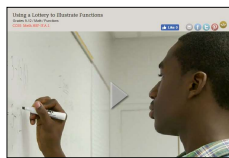
a) $m = \frac{367-112}{5-2} = \frac{255}{3} = 85$
 b) $y = 367 - 85(5) = -58$
 b) $y = 85x - 58$
 c) $y = 85(10) - 58 = 850 - 58 = \792



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Resources – Beginning Looks

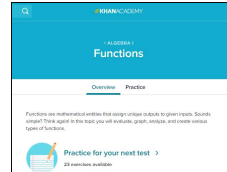


- Using a Lottery to Illustrate Functions - The Teaching Channel
https://www.teachingchannel.org/videos/teaching-functions?utm_source=Alpha+List&utm_campaign=17fa2b7690-
- Speeding Along
<http://www.floridaipdae.org/index.cfm?fuseaction=resources.GEDAHS&cagid=A37BC967EEFD18737E7AC2AF2D8421DD4A11C694934330A61EB65F4EB10E766B>
- What Are Functions? Math Antics
<https://www.youtube.com/watch?v=52tpYI2tTqk>

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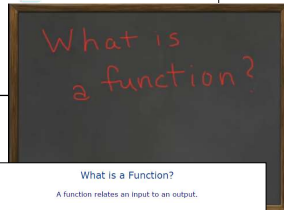


Resources



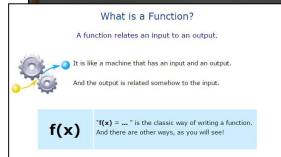
Functions – Khan Academy

<https://www.khanacademy.org/math/algebra/algebra-functions>



What is a function?

<https://www.youtube.com/watch?v=ryQJa8ybxVY>



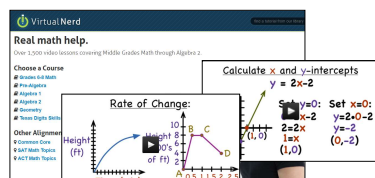
Math is Fun

<https://www.mathsisfun.com/sets/function.html>

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Resources



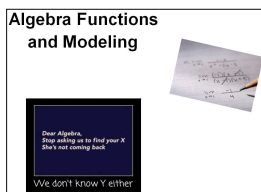
Virtual Nerd

<http://www.virtualnerd.com/>



Illuminations

<https://illuminations.nctm.org/>



Algebraic Functions and Modeling – Steve Schmidt, Appalachian State
<https://abspd.appstate.edu/node/385>

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Tips for Teaching Inequalities and Functions

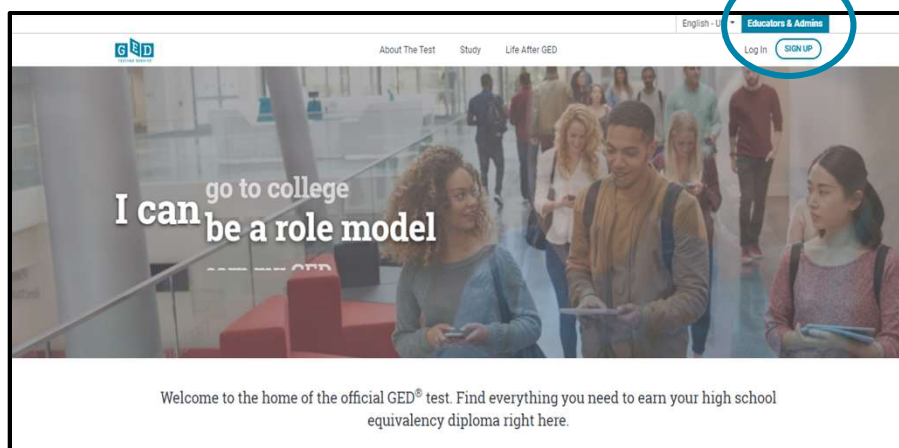


- Make it meaningful - start with concrete examples and real-world problems
- Make your thinking processes visible
- Solve the problems many ways
- Show the application
- Provide time for discourse - have students communicate their reasoning
- Ensure time for mastery of the basics

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<https://ged.com>



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