Introduction to Higher-Order Algebra for Level 1 and Level 2 Students

Resources

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The Magic of Algebra

Algebra Magic 1

- Think of a number between 1 and 100.
- Multiply your number by 4.
- Add 12.
- Multiply this number by 2.
- Add 16.
- Divide this number by 8.
- Subtract your original number.

Can you show that algebraically?

Algebra Magic 2

- Think of any number.
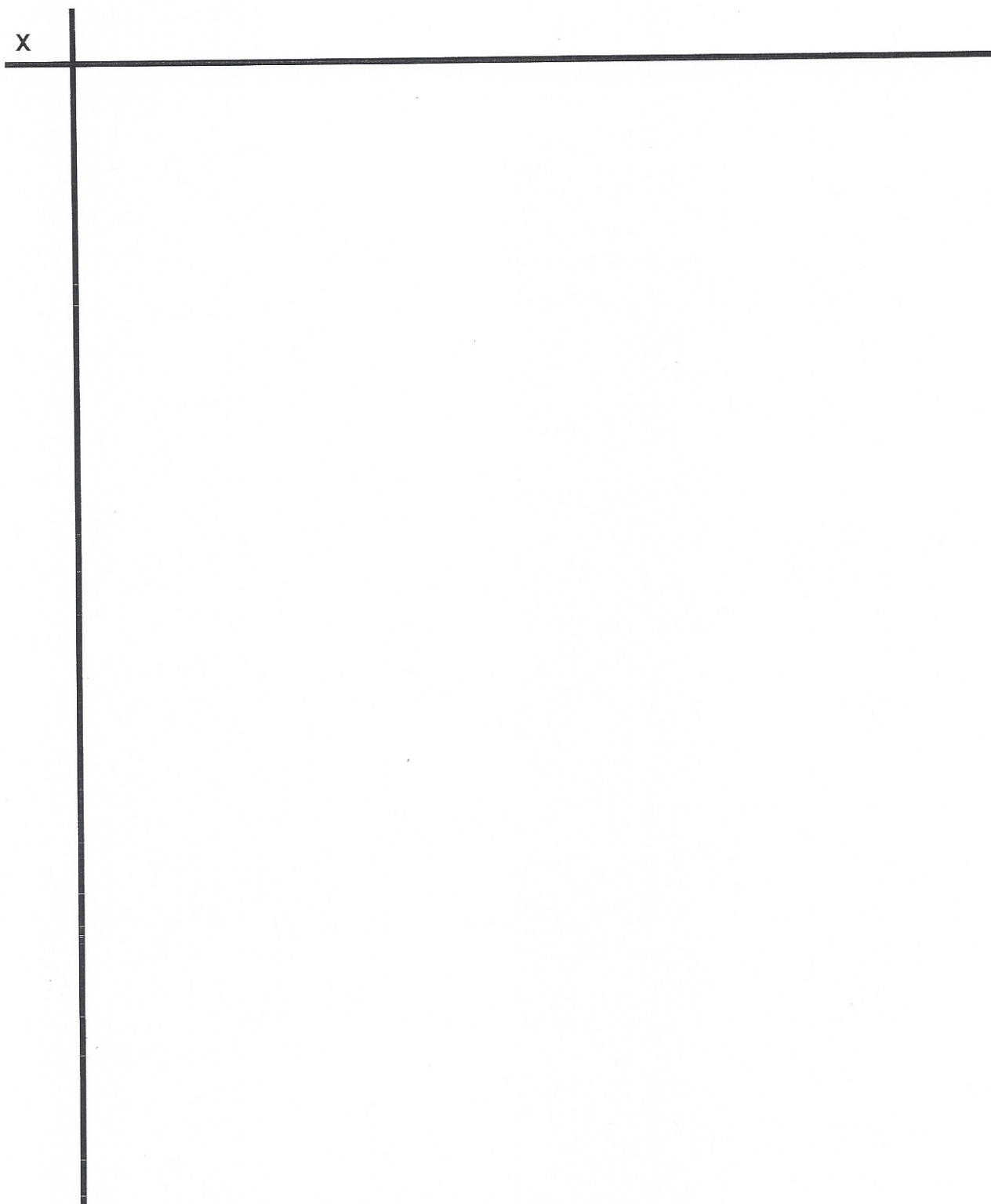- Multiply the number by 2.
- Add 4.
- Multiply by 3.
- Divide by 6.
- Subtract the number with which you started.

Explain with algebra why this works.

Algebra Tile Template

x^2	x^2	x	x	x	x
x^2	x^2	x	x	x	x
x	x	1	1	1	1
x	x	1	1	1	1
x	x	1	1	1	1
x	x	1	1	1	1

Algebra Tile Mat

X

High Impact Indicators – Algebraic Reasoning

Algebraic Reasoning		
Assessment Target	Indicators	What to look for in student work: The student can
A.3 Write, manipulate, solve, and graph linear inequalities	• **A.3.a** Solve linear inequalities in one variable with rational number coefficients. • **A.3.b** Identify or graph the solution to a one variable linear inequality on a number line. • **A.3.c** Solve real-world problems involving inequalities. • **A.3.d** Write linear inequalities in one variable to represent context.	• solve inequalities in one variable, using the standard algorithms. • solve a one-variable inequality and identified or created a graph on the number line of the solution . • analyze the relationship between quantities in a real-world problem, and then create an inequality to model the problem situation. • analyze the relationship between quantities in a real-world problem, and then solve the problem through algebraic reasoning.
A.7 Compare, represent, and evaluate functions	• **A.7.a** Compare two different proportional relationships represented in different ways. Examples include but are not limited to: compare a distance-time graph to a distance-time equation to determine which of two moving objects has a greater speed. • **A.7.b** Represent or identify a function in a table or graph as having exactly one output (one element in the range) for each input (each element in the domain). • **A.7.c** Evaluate linear and quadratic functions for values in their domain when represented using function notation. • **A.7.d** Compare properties of two linear or quadratic functions each represented in a different way (algebraically, numerically in tables, graphically or by verbal descriptions). Examples include but are not limited to: given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.	• identify functions and non-functions displayed in graphs and tables, and create functions (graphs/tables). • substitute values for variables in functions and evaluate the resulting numerical expressions. • convert functional representations from one from to another, and compare properties of the functions.

Symbols and Vocabulary for Inequalities

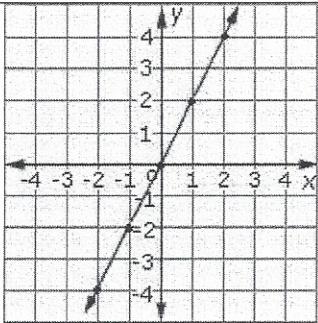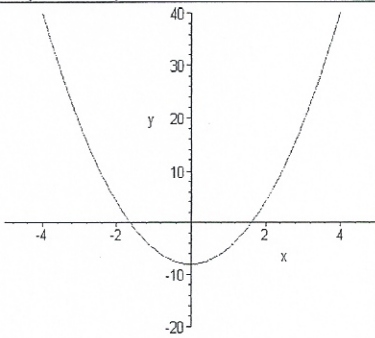| Notation or Vocabulary	Definition
$a > b$	a is more than b
$a \geq b$	a is at least b
$a < b$	a is less than b
$a \leq b$	a is at most b or a is no more than b
$a \neq b$	a is not equal to b
∞	Symbol for positive infinity – an abstract concept describing something without any bound or larger than any number.
Boundary point	A solution that makes the inequality true
Coefficient	$4a > b$ – the number associated with the variable
Inclusive	$a \leq 6$ – includes the number and is indicated on the number line with a closed circle ●
Exclusive	$A < 6$ – excludes the number and is indicated on the number line with an open circle ○
Solution Set	The range of values that make the inequality true
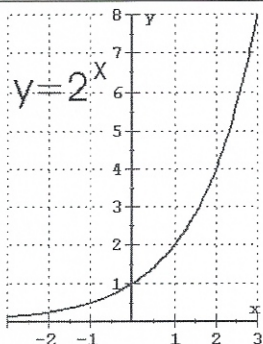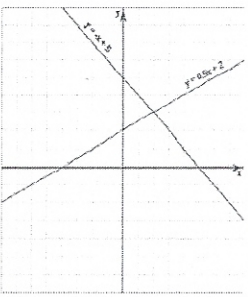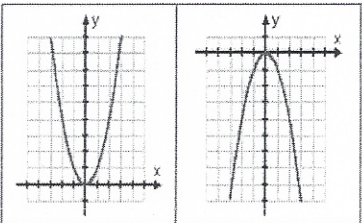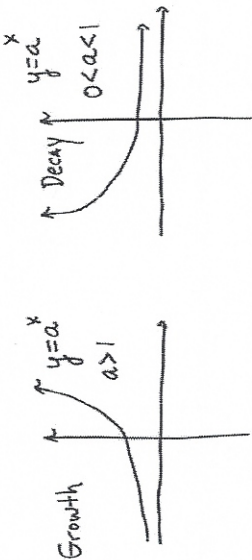
Inequality Key Words and Symbols

Symbol Æ Symbol	Meaning	Associated Words
≥	Greater than or equal to	• No less than • At least • Minimum
≤	Less than or equal to	• No more than • At most • Maximum
>	Greater than	• More than • Greater than
<	Less than	• Less than • Fewer than
=	Equal to/Equals	• The same as • Is equal to • equals

Vocabulary

domain	the set of input values or *x*-values of a function
function	a relationship between variables that has one output for each and every input
linear function	a function in which the highest power associated with the independent variable is 1
linear function	a function that is represented by a line when graphed on a Cartesian plane
range	the set of output values or *y*-values of a function
slope	a ratio of the rate at which the dependent variable is changing versus the rate at which the independent variable is changing; frequently expressed as $\frac{rise}{run}$, or $\frac{change\ in\ y}{change\ in\ x}$
y-intercept	the point on the *y*-axis at which a function crosses the *y*-axis
slope-intercept form	the form $y = mx + b$ of a linear equation, where *m* represents the slope of the line and *b* represents its *y*-intercept
absolute value	the distance a number is from zero on a number line; the value of a number ignoring its sign (+ or −)
origin	the point of intersection of the *x*-axis and *y*-axis on a Cartesian plane
Cartesian plane	a plane which has a horizontal line (x axis) and a vertical line (y axis), also known as a coordinate plane or grid

Linear, Quadratic, and Exponential Function Graphs

Type of Function	Linear	Quadratic	Exponential
Description	Straight line	U Shape (parabola) that opens up or down	Grows/Shrinks fast and levels off on 1 side
Graph			$y = 2^x$
Equation	$f(x) = ax + b$ or $f(x) = a$	$f(x) = ax^2 + bx + c$	$f(x) = a^x$
How to tell from the equation what the graph looks like:	The x term is to the first power (x)	The x term is to the second power (x^2)	The x term is the exponent (2^x)
Graph specific information:	In the equation $y = mx + b$, if m is negative, the line **decreases** (goes down) as we look from left to right: $y = -x + 5$ If m is positive, the line **increases** (goes up) as we look from left to right: $y = .5x + 2$ 	If the number with the x^2 is negative, the U shape opens down. If the number with the x^2 is positive, the U shape opens up: 	In the equation $y = a^x$, if a is larger than 1 the curve will grow fast. If a is between 0 and 1, the curve will shrink fast.

Steve Schmidt, Appalachian State University

Resources from the World Wide Web

The following are resources referenced in the workshop, as well as additional sites that expand the information.

Algebra Tiles

Working With Algebra Tiles - MathBits
http://mathbits.com/MathBits/AlgebraTiles/AlgebraTiles.htm

Factoring Polynomials Using Algebra Tiles - Del Mar College
http://dmc122011.delmar.edu/math/MLC/QEPMathSeminars/FactoringTrinomialsAlgebraTilesStudentActivity.pdf

Multiplying Polynomials Using Algebra Tiles – Virginia Dept. of Education
http://www.doe.virginia.gov/testing/solsearch/sol/math/A/m_ess_a-2b_2.pdf

Illuminations (National Council for Teachers of Math)
http://illuminations.nctm.org/activity.aspx?id=3482

Michigan Virtual University
http://media.mivu.org/mvu_pd/a4a/homework/index.html

National Library of Virtual Manipulatives
http://nlvm.usu.edu/en/nav/vlibrary.html

Inequalities

How to Graph Inequalities for Middle School: Fractions & Other Math Tips
https://www.youtube.com/watch?v=PTDN-ApjzsM

How to Solve Inequalities
https://www.youtube.com/watch?v=wYEYeFGxHkI&t=57s

One-Variable Inequalities – Khan Academy
https://www.khanacademy.org/math/algebra/one-variable-linear-inequalities

Virtual Nerds: What is an Inequality? https://www.youtube.com/watch?v=wcBwdz-ZBaM

Math is Fun – Solving Inequalities
http://www.mathsisfun.com/algebra/inequality-solving.html

Very Basics of Graphing Inequalities (on a number line)
https://www.youtube.com/watch?v=nif2PKA9bXA

Solving and Graphing Inequalities (Excellent!)
https://www.youtube.com/watch?v=EE2qWIyjKD0

Math Dude Unit 1-4 –Solving Inequalities
https://www.youtube.com/watch?v=8hhewFQ_K0w

Solving Linear Inequalities – Event Planning
www.floridaipdae.org/index.cfm?fuseaction=resources.GEDAHS&cagiid=35103C4421814CCDCF2B
F60B532270EE0718F330D6DCACE4E33EFA989573B6E6

Florida IPDAE – GED and AHS Lessons
Beginning Algebra – Lessons 14-15
http://www.floridaipdae.org/index.cfm?fuseaction=resources.GEDAHS&cagiid=DA077C783C76A85
D93EE670F44851D4C70E44B31245B6D1B60A314A7FABD6FAE

Inequalities in the Real-World
https://betterlesson.com/lesson/592219/inequalities-in-the-real-world

Inequalities – Solving and Graphing
http://alex.state.al.us/lesson_view.php?id=29038

Functions

Patterns, Functions, and Algebra – Annenberg Learner
https://www.learner.org/courses/learningmath/algebra/

Math in Practice Series from NCTM: *Putting Essential Understanding of Functions into Practice* -
Robert Ronau, Dan Meyer, Terry Crites

Using a Lottery to Illustrate Functions - The Teaching Channel
https://www.teachingchannel.org/videos/teaching-
functions?utm_source=Alpha+List&utm_campaign=17fa2b7690-

Speeding Along – A Lesson Plan from Florida IPDAE
http://www.floridaipdae.org/index.cfm?fuseaction=resources.GEDAHS&cagiid=A37BC967EEFD187
37E7AC2AF2D8421DD4A11C694934330A61EB65F4EB10E766B

Functions – Khan Academy
https://www.khanacademy.org/math/algebra/algebra-functions

What is a function? https://www.youtube.com/watch?v=ryQJa8ybxVY

Math is Fun https://www.mathsisfun.com/sets/function.html

Virtual Nerd http://www.virtualnerd.com/algebra-1/all/

Illuminations https://illuminations.nctm.org/

Algebraic Functions and Modeling – Steve Schmidt, Appalachian State
https://abspd.appstate.edu/node/385

Linear Equations, Functions, and Graphs – Khan Academy
https://www.khanacademy.org/math/algebra-home/alg-linear-eq-func

Beginning Algebra – IPDAE

http://www.floridaipdae.org/index.cfm?fuseaction=resources.GEDAHS&cagiid=DA077C783C76A85D93EE670F44851D4C70E44B31245B6D1B60A314A7FABD6FAE

What Are Functions? – Math Antics https://www.youtube.com/watch?v=52tpYl2tTqk

Insights Into Algebra 1 – The Annenberg Learner
https://www.learner.org/workshops/algebra/index.html

Properties of Functions – The Math Dude
http://www.montgomeryschoolsmd.org/departments/itv/MathDude/watch-online.aspx?id=31

Exponential Functions and Quadratic Functions – Khan Academy
https://www.khanacademy.org/math/algebra/introduction-to-exponential-functionshttps://www.khanacademy.org/math/algebra/quadratics#features-of-quadratic-functions

Inside Mathematics
http://www.insidemathematics.org/common-core-resources/mathematical-content-standards/standards-by-grade/high-school-functions

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